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# Enhancing metro network resilience via localized integration with bus services

Jian Gang Jin [a,*], Loon Ching Tang [b], Lijun Sun [c,d], Der-Horng Lee [c]

[a] School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China
[b] Department of Industrial and Systems Engineering, National University of Singapore, Singapore
[c] Department of Civil and Environmental Engineering, National University of Singapore, Singapore
[d] Future Cities Laboratory, Singapore-ETH Centre, Singapore

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ABSTRACT

This paper advances the field of network disruption analysis by introducing an application to a multi-modal transport network, capitalizing on the redundancies and improved connectivity of an integrated metro-bus network. Metro network resilience to disruptions can be enhanced by leveraging on public bus services. To ensure better acceptance among operators and commuters, we focus on introducing localized integration with bus services instead of designing an entirely new bus network to achieve the desired resilience to potential disruptions. This is accomplished by increasing the capacity of bus services that run in parallel with affected metro lines as well as those connecting to different metro lines. Our analysis starts with a network representation to model the integrated metro and bus system. A two-stage stochastic programming model is further developed to assess the intrinsic metro network resilience as well as to optimize the localized integration with bus services. The approach is applied to a case study based on the Singapore public transit system and actual travel demand data. The results show that the metro network resilience to disruptions can be enhanced significantly from localized integration with public bus services.



## 1. Introduction

Metro systems have been acting as a key solution for supporting mobility needs in high-density urban areas. Being a mode of shared transportation service, metro systems carry large quantity of commuters in a more environmentally friendly manner than private transport. The dependence on metro systems keeps growing in many cities over the world. Take Singapore as an example, a city with a population of 5 million generated around 1 million metro trips per day in 2011. Such a heavy dependence imposes enormous strains on metro systems and makes service disruptions hardly affordable. Even limited service disruptions in metro systems could result in significant productivity loss and widespread confusion. Take the 16th December 2011 disruption in Singapore's metro network for example: train services at 11 stations were disrupted for 5 h and more than 100,000 commuters were affected. Thus, the reliability of metro network and its resilience to potential disruptions should be well ensured.

Resilience of a system refers to the ability to withstand disruptions within acceptable reduction in service performance. In the context of metro systems, the resilience could be measured by the loss of capacity and the service level recovery efforts for disruption responses. Instead of relying on post-disruption recovery operations (e.g., running bus bridging services), a more effective way is to improve the intrinsic resilience so that possible disruptions (within a certain disruption scale) incur

* Corresponding author. Tel.: +86 15821519920.
*E-mail addresses:* jinjg05@gmail.com (J.G. Jin), isetlc@nus.edu.sg (L.C. Tang), sunlijun@nus.edu.sg (L. Sun), ceeleedh@nus.edu.sg (D.-H. Lee).

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**Nomenclature**

*Sets:*

| | |
|---|---|
| $W$ | set of origin–destination (OD) pairs |
| $\Omega$ | set of disruption scenarios |
| $\mathcal{N}_M$ | set of metro node |
| $\mathcal{A}_M$ | set of metro arcs |
| $\mathcal{N}_B$ | set of bus node |
| $\mathcal{A}_B$ | set of bus arcs, union of parallel bus arc set $\mathcal{A}_B^1$ and inter-line bus arc set $\mathcal{A}_B^2$ |
| $\mathcal{A}_T$ | set of transfer arcs |
| $\mathcal{N}$ | $\mathcal{N} = \mathcal{N}_M \cup \mathcal{N}_B$ |
| $\mathcal{A}$ | $\mathcal{A} = \mathcal{A}_M \cup \mathcal{A}_B \cup \mathcal{A}_T$ |
| $K_w$ | set of feasible paths in the metro-bus network for OD pair $w$ |
| $B$ | set of bus lines |
| $P_b$ | set of localized integration plans for a certain bus line $b$ |
| $\Psi_b$ | $\Psi_b := P_b \times P_b$, set of integration plans that cannot be introduced simultaneously for bus line $b$ |

*Parameters:*

| | |
|---|---|
| $\zeta$ | the resilience of a metro network |
| $d_w$ | amount of travel demand that is satisfied under disruption condition between OD pair $w$ |
| $D_w$ | total travel demand for OD pair $w$ under normal condition |
| $(\rho_i^1, \rho_i^2)$ | representation of a node with $\rho_i^1$ and $\rho_i^2$ indicating the metro station and travel mode, respectively |
| $c_{ij}$ | service capacity (maximum number of commuters per hour) of arc $(i,j) \in \mathcal{A}$ |
| $t_{ij}$ | travel/transfer time of arc $(i,j) \in \mathcal{A}$ |
| $(u_w, v_w)$ | origin and destination nodes of OD pair $w$ |
| $\beta_w$ | maximum travel time increase allowed for OD pair $w$ |
| $T_w^0$ | journey time of OD pair $w$ when no disruption occurs |
| $c_b$ | the hourly spare service capacity of bus line $b$ that can be used by metro commuters during disruptions |
| $\theta_{ij}^b$ | 1 if bus arc $(i,j)$ is covered by the current bus line $b$; and 0 otherwise |
| $\delta_{pij}$ | 1 if bus arc $(i,j)$ is newly covered in the integration plan $p$; and 0 otherwise |
| $\delta_{pij'}$ | 1 if the currently covered bus arc $(i,j)$ is removed in the integration plan $p$; and 0 otherwise |
| $\Delta c_{ij}(\xi)$ | the capacity reduction on arc $(i,j) \in \mathcal{A}_M$ under disruption scenario $\xi \in \Omega$ |
| $\gamma_{wij}^k$ | 1 if arc $(i,j)$ is used by the $k$th shortest path of OD pair $w$; and 0 otherwise |
| $q_i^1$ | the commuter in-flow capacity for station $i$ |
| $q_i^2$ | the commuter out-flow capacity for station $i$ |
| $\Delta q_i^1(\xi)$ | the reduction of metro node in-flow capacity under disruption scenario $\xi$ for station $i$ |
| $\Delta q_i^2(\xi)$ | the reduction of metro node out-flow capacity under disruption scenario $\xi$ for station $i$ |
| $Z(\xi)$ | the fraction of travel demand fulfillment under disruption scenario $\xi$ |
| $L_1^b$ | maximum number of employed integration adjustments for bus line $b$ |
| $q_p^b$ | additional number of buses needed if localized integration plan $p$ for bus line $b$ is introduced |
| $L_2$ | number of additional buses available |
| $\varphi_{wik}^1$ | 1 if commuter flow $w$ on path $k$ belongs to the in-flow of the metro station $i$; and 0 otherwise |
| $\varphi_{wik}^2$ | 1 if commuter flow $w$ on path $k$ belongs to the out-flow of the metro station $i$; and 0 otherwise |
| $p_\xi$ | the weight of disruption scenario $\xi$ |

*Decision variables:*

| | |
|---|---|
| $x_{ij}^k$ | 1, if arc $(i,j)$ belongs to the $k$th shortest path; 0, otherwise |
| $y_p^b$ | 1, if the localized integration plan $p \in P_b$ of bus line $b$ is selected; 0, otherwise |
| $f_w^k(\xi)$ | $\geqslant 0$. The commuter flow of OD pair $w$ on path $k$ under disruption scenario $\xi$ |

minimum performance reduction or even no negative impact ideally. Measures of improving metro network resilience include (1) building a well-connected network with self-adaptive ability to recover from disruptions, and (2) integrating metro and bus systems in such a way that bus system provides as much backup capacity as possible during metro disruptions. The former measure usually takes a long-term horizon, while the latter is an option for resilience improvement before the well-connected metro network is fully constructed.

In this paper, we take the measure of leveraging on public bus services to improve the metro system's resilience to disruptions. Note that this study addresses the response planning for metro system disruptions with the domain of facilitating those "business-as-usual" travelers, as opposed to emergency responders or evacuees. System level integration between metro and bus systems is capable of complementing each other when service disruption occurs in one system. However, establishing integration between the two systems often comes at the cost of complicated planning and commuters' dissatisfaction due to travel practice changes. With this regard, we focus on introducing localized integration with bus services instead of designing an entirely new bus network in order to enhance the metro network resilience to potential disruptions. The basic idea is to introduce incremental adjustments on current bus services and enhance the localized integration between metro system and bus services in such a way that commuters can be directed to alternative pathways either by bus services or the degraded metro system. In other words, we improve the complementary capacity of bus services to metro network by optimizing the connectivity between the two systems. The contribution of this study lies in the following aspects:

- Propose a quantitative assessment for the metro system resilience;
- Develop a mathematical modeling framework for optimizing localized integration between metro and bus systems. Such a pro-active management strategy aims to improve the metro network resilience while incurring minimal modifications to public bus services. The modeling framework is also capable of assessing metro system's intrinsic ability to withstand potential service disruption;
- Demonstrate the practical significance of the proposed method. A case study based on the Singapore public transit system shows that the metro network resilience to disruptions can be improved considerably by the proposed localized integration between metro and bus systems.

This remainder of the paper is organized as follows. Section 2 reviews relevant papers in the literature and highlights the research gap. Section 3 develops the a modeling framework for enhancing the metro network resilience. A case study based on a real-world metro system is conducted in Section 4. Finally, Section 5 draws conclusions and suggests future research directions.

## 2. Literature review

As two major layers of the urban transit system, metro and bus services need to be well integrated. This issue has received much attention in the literature on urban transit system design and optimization. A well-acknowledged approach is the so-called hub-and-spoke network, in which metro lines connect areas with high travel demand while feeder bus lines further expand public transit service to other areas. Kuah and Perl (1989) first defined the feeder-bus network-design problem given an existing metro system. Spasovic et al. (1994) proposed a framework to find the optimal bus service coverage in urban corridor based on social welfare maximization. Moorthy (1997) proposed an integrated approach for the planning and evaluation of urban mass transport systems with both bus and light/rapid transit services. Lin and Chen (2008) developed a generalized hub-and-spoke network design framework incorporating three featured hub-and-spoke networks with the objective of minimizing operational costs. Li et al. (2009) proposed analytical models to optimize a metro system with feeder bus services under different market regimes, such as profit maximization and social welfare maximization. Other than the strategic planning and operational management for transit networks, the issue of coordination between multi-level transit systems should also be well addressed (Guihaire and Hao, 2008). Sivakumaran et al. (2012) introduced the concept of dispatching coordination between feeder and trunk services in a public transit system and showed great benefit for both transit operators and users.

One critical consideration involved in transit network design and optimization is the resilience of the public transport system, i.e., the ability to withstand potential disruptions. This topic has been receiving more and more attention recently, since transit systems have become so vital to urban mobility that we can hardly bear severe service breakdown. Unfortunately trains do not always run on time due to unexpected events (e.g., infrastructure malfunctions, accidents and extreme weather conditions), and these disruptions have become more frequent and severe, such as the recent examples occurred in Singapore (Pender et al., 2012). Two directions of dealing with transit system's disruptions are as follows:

(1) *Post-disruption response/recovery*: Post-disruption response focuses on devising responsive measures for transportation system disruptions in order to alleviate their consequences. Meyer and Belobaba (1982) examined the contingency planning processes for urban transportation disruptions. Cadarso et al. (2013) studied the disruption recovery problem of rapid transit rail networks. An integrated optimization model for determining timetables and rolling stock schedules in the residual rail network was developed, with a specific consideration of passengers' behavior under disruptions. Jin et al. (2013) proposed a different recovery approach for metro system disruptions in which temporary shuttle bus services are provided in the affected area in order to complement the degraded metro system. Shuttle bus routes were generated dynamically by a column generation procedure, and the best combination of bus routes was selected via a path-based multi-commodity flow model. Kepaptsoglou and Karlaftis (2009) studied the same disruption recovery problem (termed

as bus bridging problem in their paper), and developed a modeling framework with three hierarchical steps: definition of bus bridging environment, design of bus routes, and allocation of resources. As a different post-disruption recovery measure, having taxis instead of buses as the recovery service for on-board passengers in a public tram system was studied by Zeng et al. (2012). The authors formulated tram and taxi companies' decision functions in order to support the collaboration between the two parties as well as the service compensation scheme.

(2) *Pre-disruption preparedness*: Another way of dealing with metro system disruptions is to prepare certain measures before disruption happens, as is referred as pre-disruption preparedness. The objective is to improve the resilience of the metro system in such a way that potential disruptions incur minimal negative impact. Similar research topics can be identified in the area of more general transportation networks, such as road network (Liu et al., 2009) and freight transportation networks (Chen and Miller-Hooks, 2012; Miller-Hooks et al., 2012). Liu et al. (2009) examined the problem of allocating limited retrofit resources over multiple highway bridges with the objective of improving the resilience and robustness of the entire transportation network. Chen and Miller-Hooks (2012) defined an indicator of network resilience as the ability of an intermodal freight transport network to recover from natural or human-caused disruptions. A stochastic mixed-integer programming model was developed for evaluating network resilience, together with the decision of post-event recovery actions. Based on the same resilience definition, Miller-Hooks et al. (2012) further incorporated pre-disruption preparedness decision and developed a more general model.

To the best of our knowledge, in the existing literature little attention was paid to disruption impact when designing urban transit systems (metro and bus) and very few researchers studied the pre-disruption preparedness for metro system disruptions. Aiming at filling this research gap, this paper introduces the approach to enhancing metro network resilience via localized integration with bus services, and develops a mathematical modeling framework to find the best metro-bus integration plan.

## 3. Modeling framework

In this section, we first introduce a quantitative index for metro network resilience and give a detailed problem description. Then, a modeling framework with three steps is presented: (1) network representation for the integrated metro and bus system; (2) generation of alternative paths under disruptive conditions; and (3) localized integration with bus services.

### 3.1. Problem description

Mathematically, the resilience of a metro network $\zeta$ can be expressed as the fraction of travel demand that can be satisfied by the degraded metro network after disruptions:

$$\zeta = \sum_{w \in W} d_w \Big/ \sum_{w \in W} D_w \tag{1}$$

where $W$ is the set of origin–destination (OD) pairs, $d_w$ is the amount of travel demand that is satisfied within a certain time window under disruption between the OD pair $w$, and $D_w$ is the total demand for the OD pair under normal condition. Note that the network resilience definition here is the same as that of Chen and Miller-Hooks (2012), except that they defined the resilience for intermodal freight transport networks while we focus on metro system. We remark that metro and freight transport networks share analogous characteristics in terms of network topology and functions, and the above definition is able to reflect network's residual capability of fulfilling demands. One may argue to measure metro network resilience by the change in the aggregated travel time, as it is also a proxy for disruption cost. However, during metro network disruptions, transport authorities are more concerned about whether or not the affected commuters can be directed to alternative paths and get to their destinations, while minimizing the travel time increase may not be the first priority.

A well-planned bus service system is able to complement metro system during normal condition and in particular when metro service disruption occurs. This is because the bus services could satisfy partial travel demand that cannot be fulfilled by the degraded metro network. Here we assume that the disruptions only happen to the metro system while bus services function normally under disruption. Cases could be a fire incident that causes a metro station together with the metro services with neighboring stations to shut down, or a loss-of-power incident that causes service suspension along a partial metro line. It could also be a case where an unexpected mechanical fault increases train interval and reduces service capacity. In such disruptive cases, bus services could be employed as the complementary system to serve those affected metro commuters. Note that those disruptions affecting both of metro and bus systems (e.g., hurricane) are out of the scope of this study since bus services can no longer complement the metro system. The backup capacity of the bus system is determined by the following two types of bus services (as shown in Fig. 1):

(1) *Parallel bus service*: those running in parallel with metro lines and providing backup capacity of carrying commuters when corresponding metro links break down;
(2) *Inter-line bus service*: those connecting different metro lines and directing commuters swiftly to alternative pathways.

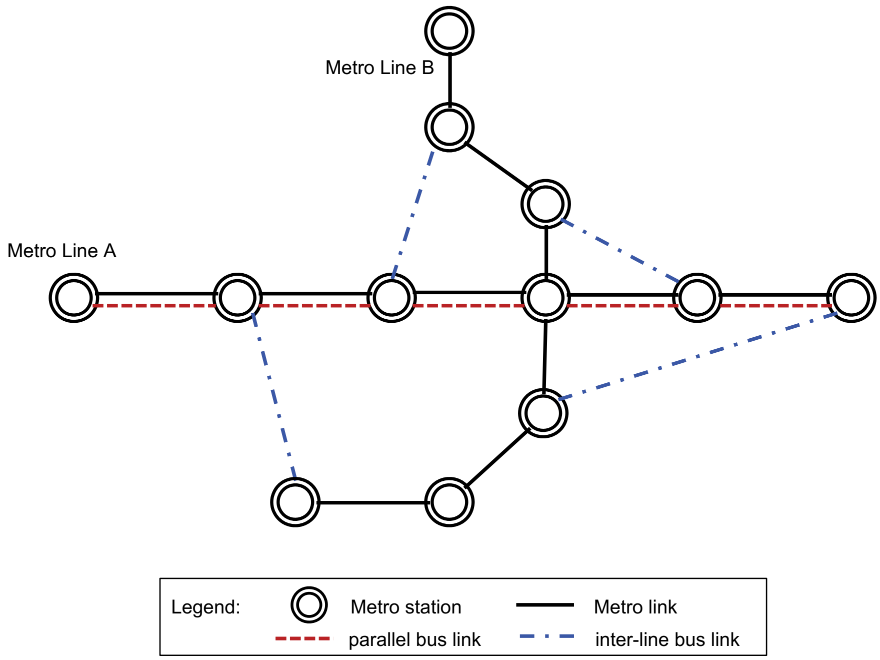


**Fig. 1.** An illustrative example of the parallel and inter-line bus services.

In order to fully utilize the complimentary capacity of bus services during metro disruptions, the integration between the two travel modes (i.e., metro and bus) should be implemented in such a way that travel demand could be satisfied by either the degraded metro system or bus services to the greatest extent. In this regard, we introduce localized integration to those existing bus services which are not well connected with metro systems. Taking bus service No. 23 in Singapore (shown in Fig. 2) as an example, it covers the service area between two metro lines (EW and NE). By introducing localized integration to the route as indicated by the dash-dot line, it could contribute to the backup capacity of the parallel bus service (EW11-EW12, NE7-NE8) as well as inter-line bus service (EW12-NE7).

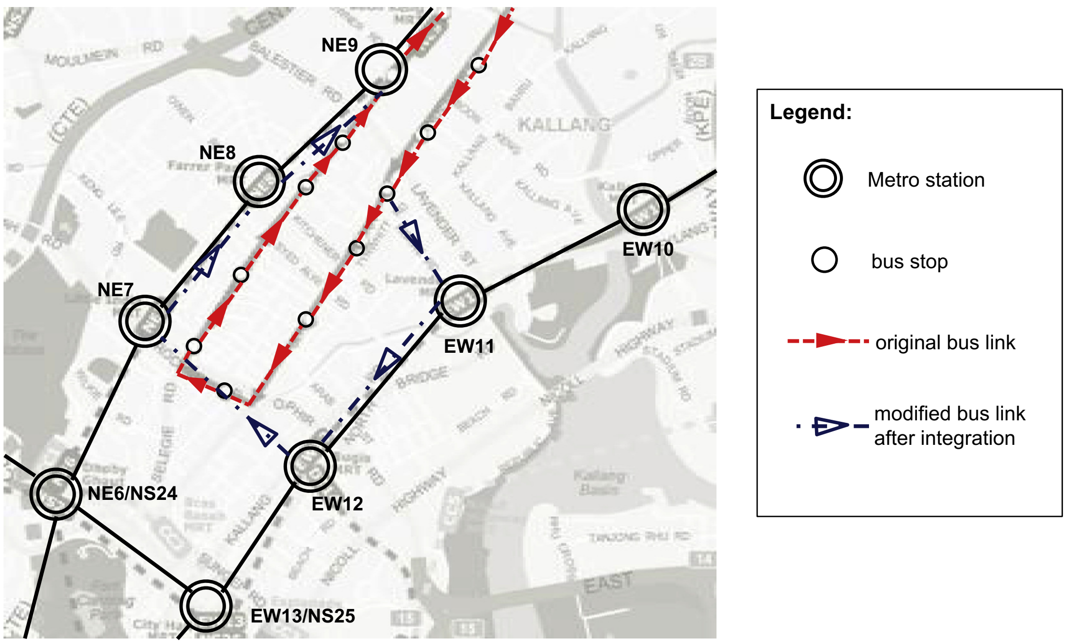


**Fig. 2.** Route of bus service No. 23 in Singapore.

Given the metro network and existing bus services, the challenge of the metro network resilience optimization problem is to smartly introduce localized integration between metro and bus systems in order to improve the capacity of the parallel and inter-line bus services. Formally, the problem is to construct an integrated metro-bus service network by optimally selecting localized integration plans subject to certain constraints so that maximum travel demand can be satisfied under metro network disruptions.

Considering that disruptions in metro networks are highly uncertain, the metro network resilience could be measured as the expectation of the demand fulfillment over a set of pre-defined disruption scenarios $\Omega$:

$$\max \mathbb{E}_{\xi\in\Omega}\left\{\sum_{w\in W} d_w(\xi) \Big/ \sum_{w\in W} D_w(\xi)\right\} \tag{2}$$

where $d_w(\xi)$ and $D_w(\xi)$ represent the fulfilled travel demand under disruption scenario $\xi$ and the original travel demand for OD pair $w$, respectively.

### 3.2. Network representation

In order to model commuter traveling and transferring between the two travel modes, the integrated metro-bus network is represented as a graph, as illustrated by Fig. 3. Each node $i$ is represented as a tuple $(\rho_i^1, \rho_i^2)$ where $\rho_i^1$ and $\rho_i^2$ indicate the metro station and travel mode, respectively. Thus, each metro station is associated with two nodes: one metro node and one bus node. We then define the metro system as a directed graph $\mathcal{G}_M(\mathcal{N}_M, \mathcal{A}_M)$ where $\mathcal{N}_M$ is the metro node set and $\mathcal{A}_M$ is the metro arc set. Similarly, the bus service network can be defined as another directed graph $\mathcal{G}_B(\mathcal{N}_B, \mathcal{A}_B)$ where $\mathcal{N}_B$ is the set of bus nodes that pair with metro nodes, and the bus service arc set $\mathcal{A}_B$ consists of two subsets: parallel bus arcs $\mathcal{A}_B^1$ and inter-line bus arcs $\mathcal{A}_B^2$. The parallel bus arcs link neighboring metro stations in the same way as the metro arcs, while the inter-line bus arcs connect metro stations on different lines. Note that the inter-line bus arcs should be defined for those connections where inter-line bus services already exist or should be considered to deploy. For a certain station, the associated metro node and the bus node are connected via two opposite-directed transfer arcs, as shown by Fig. 3. Let $\mathcal{A}_T$ represent the set of those transfer arcs. Further define node set $\mathcal{N} = \mathcal{N}_M \cup \mathcal{N}_B$, arc set $\mathcal{A} = \mathcal{A}_M \cup \mathcal{A}_B \cup \mathcal{A}_T$ and an overall directed graph $\mathcal{G}(\mathcal{N}, \mathcal{A})$.

### 3.3. Alternative path generation

Based on the defined integrated metro-bus network, alternative paths $K_w$ that can be possibly used by OD pair $w \in W$ under disruptive conditions should be generated. In this paper, we consider the $k$ shortest paths (Yen, 1971). Different from the algorithm by Yen (1971), we formulate a series of integer programs that are customized to the integrated metro-bus network, and take the advantage of advanced optimization solvers (e.g., CPLEX) to obtain the path set efficiently. Define parameter $c_{ij}$ as the service capacity (maximum number of commuters per hour) of arc $(i,j) \in \mathcal{A}$ and $t_{ij}$ as the travel/transfer time of arc $(i,j) \in \mathcal{A}$. The origin and destination nodes of OD pair $w \in W$ are represented as $u_w$ and $v_w$, respectively. Define binary decision variable $x_{ij}^k$ be 1 if arc $(i,j) \in \mathcal{A}$ belongs to the $k$th shortest path; and 0 otherwise. Then, the $k$th shortest path generation for OD pair $w \in W$ can be formulated as the following integer program [**PGM (w,k)**].

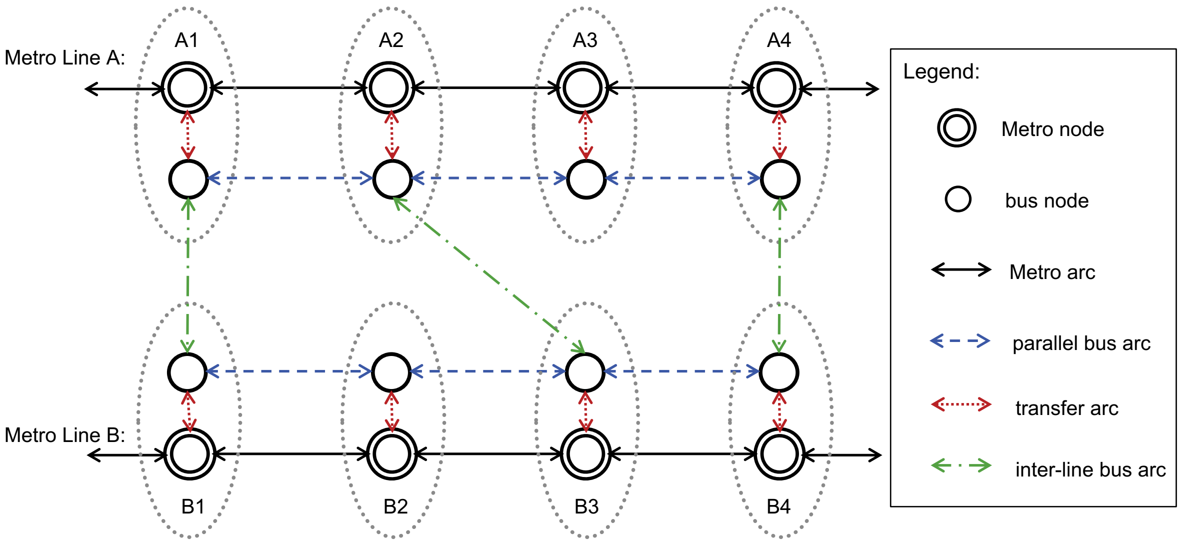


**Fig. 3.** An illustrative example of the integrated metro-bus network definition.

$$[\mathbf{PGM}(\mathbf{w},\mathbf{k})] \quad \text{minimize} \quad \sum_{(i,j)\in\mathcal{A}} t_{ij}x_{ij}^k \tag{3}$$

$$\text{subject to} \quad \sum_{j\in\mathcal{N}|(u_w,j)\in\mathcal{A}} x_{u_wj}^k = 1 \tag{4}$$

$$\sum_{j\in\mathcal{N}|(j,u_w)\in\mathcal{A}} x_{ju_w}^k = 0 \tag{5}$$

$$\sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij}^k - \sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ji}^k = 0 \quad \forall i\in\mathcal{N}|i\neq u_w, v_w \tag{6}$$

$$\sum_{i\in\mathcal{N}|(i,v_w)\in\mathcal{A}} x_{iv_w}^k = 1 \tag{7}$$

$$\sum_{i\in\mathcal{N}|(v_w,i)\in\mathcal{A}} x_{v_wi}^k = 0 \tag{8}$$

$$x_{ij}^k \in \{0,1\} \quad \forall (i,j)\in\mathcal{A} \tag{9}$$

Objective function (3) minimizes the total travel time from the origin node to the destination node for OD pair $w \in W$. Constraints (4) and (5) define the flow restriction for the origin node while Constraints (7) and (8) impose similar requirement for the destination node. The flow conservation constraints for other nodes are specified by (6).

We first set $k = 1$ and solve problems [**PGM (w,1)**] to find the shortest path for each OD pair $w \in W$. In order to find the $k$th ($k \geqslant 2$) shortest path, the following cuts are further added in the model in order to avoid the previous ($k - 1$) paths:

$$\sum_{(i,j)\in\mathcal{A}|x_{ij}^{l,*}=1} \left(1 - x_{ij}^k\right) \geqslant 1 \quad \forall l|1 \leqslant l < k \tag{10}$$

where $\{x_{ij}^{l,*}\}$ represents the optimal solution to $l$th shortest path model.

The following constraints are further imposed to ensure that the generated paths are realistic. Constraint (11) guarantees that the journey time of the $k$th path for OD pair $w$ under disruptive condition does not increase by a maximum limit $\beta_w$ compared with the original travel time under normal condition $T_w^0$. Constraint (12) ensures that the number of inter-modal transfers between metro and bus modes should not exceed a certain limit, which is set as 2 in this study. Note that this parameter could be related with the metro network size and the disruption scale, and should be calibrated beforehand. Constraints (13) and (14) further impose that each node is visited at most once in order to eliminate sub-tours. The iterative generation of shortest paths terminates until no further path satisfying Constraint (11) exists.

$$\sum_{(i,j)\in\mathcal{A}} t_{ij}x_{ij} \leqslant T_w^0 + \beta_w \tag{11}$$

$$\sum_{(i,j)\in\mathcal{A}_T} x_{ij} \leqslant 2 \tag{12}$$

$$\sum_{j\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij} \leqslant 1 \quad \forall i\in\mathcal{N} \tag{13}$$

$$\sum_{i\in\mathcal{N}|(i,j)\in\mathcal{A}} x_{ij} \leqslant 1 \quad \forall j\in\mathcal{N} \tag{14}$$

### *3.4. Localized integration with bus services*

The metro network resilience optimization model can be formulated as a two-stage stochastic mixed integer program with the key decisions of introducing localized integration with bus services. Let $B$ denote the set of bus lines, and $c_b$ be the hourly *spare* service capacity of bus line $b \in B$ that can be used by metro commuters during disruptions. Note that the existing bus-specific demand itself should be accounted when setting the parameter $c_b$, since those bus lines with high travel demand do not have much spare backup capacity for metro commuters during disruptions. Define parameter $\theta_{ij}^b$ be 1 if bus arc $(i,j) \in \mathcal{A}_B$ is covered by the current bus line $b \in B$; and 0 otherwise. The localized integration plans for a certain bus line $b \in B$ are represented as a set $P_b$, in which each plan $p$ specifies the additional covered bus arcs as well as the removed bus arcs due to the integration adjustment. Let parameter $\delta_{pij}$ be 1 if bus arc $(i,j)$ is newly covered in the integration plan $p$, and 0 otherwise. Further let parameter $\delta'_{pij}$ be 1 if the currently covered bus arc $(i,j)$ is removed in the integration plan $p$, and 0 otherwise.

Therefore, the key decision is to select a subset of localized integration plans from the set $P_b$ for each bus line $b \in B$ so that maximum commuters could be served under disruptions either by the degraded metro system or the bus services.

In order to account for the impact of disruptions on the metro arc capacity, we define parameter $\Delta c_{ij}(\xi)$ as the capacity reduction on arc $(i,j) \in \mathcal{A}_M$ under disruption scenario $\xi \in \Omega$. Let binary parameter $\gamma^k_{wij}$ be 1 if and only if arc $(i,j) \in \mathcal{A}$ is used by the $k$th shortest path of OD pair $w \in W$. Considering that metro nodes may also get affected (e.g., entrance closure) in certain disruption cases, we define $q^1_i$ and $q^2_i$ as the commuter in-flow and out-flow capacity for station $i \in \mathcal{N}_M$, respectively. The reductions of the in-flow and out-flow capacity of metro nodes under disruption scenario $\xi \in \Omega$ are represented by parameter $\Delta q^1_i(\xi)$ and $\Delta q^2_i(\xi)$, respectively.

The following decision variables are defined:

- $y^b_p$: $\in \{0,1\}$. 1 if the localized integration plan $p \in P_b$ of bus line $b \in B$ is selected; 0 otherwise.
- $f^k_w(\xi)$: $\geqslant 0$. The commuter flow of OD pair $w \in W$ on path $k \in K_w$ under disruption scenario $\xi \in \Omega$.

Then, the localized integration step could be formulated as the following two-stage stochastic program:

*Stage 1:*

$$\text{maximize} \quad \mathbb{E}_\xi\{Z(\xi)\} \tag{15}$$

$$\text{subject to} \quad \sum_{p \in P_b} y^b_p \leqslant L^b_1 \quad \forall b \in B \tag{16}$$

$$\sum_{b \in B}\sum_{p \in P_b} q^b_p y^b_p \leqslant L_2 \tag{17}$$

$$y^b_{p_1} + y^b_{p_2} \leqslant 1 \quad \forall b \in B, \forall (p_1, p_2) \in \Psi_b \tag{18}$$

$$y^b_p \in \{0,1\} \quad \forall b \in B, \forall p \in P_b \tag{19}$$

The first stage is to maximize the expectation of the fraction of satisfied travel demand $Z(\xi)$ over all disruption scenarios, as expressed by (15). Two operational restrictions are considered in the first stage as the budget for localized integration with bus services. First, since the introduction of integration adjustment to existing bus services may incur additional cost (e.g., longer round-trip time, dissatisfaction by affected commuters), the total number of employed integration adjustments for bus line $b$ should not exceed a certain limit $L^b_1$, as ensured by Constraint (16). This is to avoid significant changes on the existing bus lines so as to limit the degree of commuters' dissatisfaction. Second, considering that localized integration plan $p$ for bus service $b$ requires $q^b_p$ additional buses, we impose the overall bus resource budget restriction by Constraint (17) stating that the total number of additional buses for all integration adjustments does not exceed $L_2$. Note that the parameter $q^b_p$ could be negative as it is possible to reduce the number of buses due to integration adjustment. Besides, we define set $\Psi_b := P_b \times P_b$ to include those integration plans that cannot be introduced simultaneously for bus line $b \in B$. Constraint (18) ensures that at most one plan could be selected from those conflicting integration plans.

*Stage 2:*

$$Z(\xi) \quad = \text{maximize} \quad \sum_{w \in W}\sum_{k \in K_w} f^k_w(\xi) \Big/ \sum_{w \in W} D_w(\xi) \tag{20}$$

$$\text{subject to} \quad \sum_{k \in K_w} f^k_w(\xi) \leqslant D_w(\xi) \quad \forall w \in W \tag{21}$$

$$\sum_{w \in W}\sum_{k \in K_w} \gamma^k_{wij} f^k_w(\xi) \leqslant c_{ij} - \Delta c_{ij}(\xi) \quad \forall (i,j) \in \mathcal{A}_M \tag{22}$$

$$\sum_{w \in W}\sum_{k \in K_w} \gamma^k_{wij} f^k_w(\xi) \leqslant \sum_{b \in B} c_b \left[\theta^b_{ij} + \sum_{p \in P_b} (\delta_{pij} - \delta_{pij'}) y^b_p\right] \quad \forall (i,j) \in \mathcal{A}_B \tag{23}$$

$$\sum_{w \in W | v_w = i}\sum_{k \in K_w} \varphi^1_{wik} f^k_w(\xi) \leqslant q^1_i - \Delta q^1_i(\xi) \quad \forall i \in \mathcal{N}_M \tag{24}$$

$$\sum_{w \in W | u_w = i}\sum_{k \in K_w} \varphi^2_{wik} f^k_w(\xi) \leqslant q^2_i - \Delta q^2_i(\xi) \quad \forall i \in \mathcal{N}_M \tag{25}$$

$$f_w^k(\xi) \geqslant 0 \quad \forall w \in W, \forall k \in K_w \tag{26}$$

Given the localized integration decision $y_p^b$, the second stage is to assign, for each disruption scenario, the commuter flow on the corresponding shortest paths in such a way that maximum demand fulfillment $Z(\xi)$ could be achieved, as expressed by (20). Constraint (21) guarantees that the total commuter flow along all paths between a particular OD pair $w \in W$ under disruption scenario $\xi$ does not exceed the original travel demand. To account for the disruption impact on the metro network (i.e., the capacity of metro arcs $\mathcal{A}_M$ reduces by $\Delta c_{ij}(\xi)$), the post-disruption commuter flow $\sum_{w \in W} \sum_{k \in K_w} \gamma_{wij}^k f_w^k(\xi)$ on metro arc $(i,j)$ should not exceed the degraded capacity $c_{ij} - \Delta c_{ij}(\xi)$, as ensured by Constraint (22). Similarly, Constraint (23) guarantees that the commuter flow on each bus arc should respect the total service capacity of adjusted bus lines that pass through the particular bus arc. On the right-hand-side of Constraint (23), the component $\theta_{ij}^b + \sum_{p \in P_b} (\delta_{pij} - \delta_{pij'}) y_p^b$ takes 1 if the bus arc $(i,j)$ is covered by bus line $b$ after integration adjustment, and takes 0 otherwise. Constraints (24) and (25) are further defined to account for the disruption impact on the in-flow and out-flow capacity of individual metro stations. Constraint (24) ensures that the total number of commuters arriving at station $i$ by metro does not exceed the in-flow capacity $q_i^1 - \Delta q_i^1(\xi)$ under disruption, where parameter $\varphi_{wik}^1$ takes 1 if commuter flow $w \in W$ on path $k \in K_w$ belongs to the in-flow of the metro station $i$. Similarly, the out-flow restriction is imposed by Constraint (25) where parameter $\varphi_{wik}^2$ takes 1 if commuter flow $w \in W$ on path $k \in K_w$ belongs to the out-flow of metro station $i$.

The disruption scenarios are represented by a discrete set $\Omega$, in which each scenario $\xi$ is associated with a certain weight $p_\xi$. The weight of each scenario could be set according to the relative occurrence probability and the magnitude of disruption impacts. With the introduction of the weight parameters, the two objective functions of the two stages could be combined as a single objective function:

$$\text{maximize} \quad \left\{ \sum_{\xi \in \Omega} \sum_{w \in W} \sum_{k \in K_w} p_\xi f_w^k(\xi) \bigg/ \sum_{w \in W} D_w(\xi) \right\} \tag{27}$$

### *3.5. Non-optimized metro network resilience*

The above developed optimization model, not only enables us to enhance the metro network resilience by introducing localized integration between metro and bus systems, but also can be employed to assess the intrinsic resilience of the metro network without bus services, as well as the backup capacity of the existing bus services.

In order to obtain the metro network resilience with the existing bus services, the following constraint can be added in the model:

$$\sum_{b \in B} \sum_{p \in P_b} y_p = 0 \tag{28}$$

In order to obtain the intrinsic resilience of the metro network, we can simply further impose zero capacity restriction on all bus arcs and remove transfers and bus nodes, and solve the resulting model:

$$\sum_{w \in W} \sum_{k \in K_w} \gamma_{wij}^k f_w^k(\xi) = 0 \quad \forall (i,j) \in \mathcal{A}_B \tag{29}$$

## 4. Case study

In this section we apply the proposed modeling framework to optimize the localized metro-bus integration for the public transportation system in the central area of the city of Singapore, as shown in Fig. 4. Section 4.1 presents the main features of the case study. Section 4.2 reports the results of the metro resilience improvement via localized integration with bus services. The impact of travel demand is analyzed in Section 4.3. Section 4.4 discusses how to determine the optimal bus fleet size. Section 4.5 assesses the contribution of different types of bus services on the metro network resilience improvement, and also discusses some real-life insights.

### *4.1. Main features of the case study*

The case study area covers Singapore's central business district and the neighboring districts with high commuting demand. Currently, the public transportation system in the area consists of 61 bus lines and four intertwined metro lines (EW, NE, NS, CC) with 20 stations. We remark that this case study is representative of those high density metro systems commonly utilized by large cities like Shanghai, London and Tokyo. Thanks to the automatic fair collection system recording the origin, destination and timestamp information for all personnel trips, the commuters' travel demand can be easily obtained from smart card data. For the existing 61 bus lines passing through this area, we identify 57 candidate localized metro-bus integration plans within the study area. In order to account for the impact of different disruption scales, five scenario groups are considered (see Table 1). Each disruption scenario is characterized by a certain number of consecutive disruptive stations


**Fig. 4.** Singapore metro network in central area.

**Table 1**
Description of disruption scenarios.

| Disruption scale | Scenario description | No. of scenarios |
|---|---|---|
| A | 6 consecutive stations along a line & 10 connecting arcs affected | 5 |
| B | 5 consecutive stations along a line & 8 connecting arcs affected | 9 |
| C | 4 consecutive stations along a line & 6 connecting arcs affected | 13 |
| D | 3 consecutive stations along a line & 4 connecting arcs affected | 17 |
| E | 2 consecutive stations along a line & 2 connecting arcs affected | 21 |

along a metro line. Those metro arcs between the disruptive stations are also affected and assumed to carry zero service capacity. All the possible disruption scenarios of the metro network within the study area are enumerated and considered in the case study. Note that the disruption of bus lines is not considered in this study.

The parameters related with the operational restrictions of bus routes are set as follows:

- Maximum travel time increase during disruptions: $\beta_w = 20min, \forall w \in W$;
- Maximum number of localized integration adjustment for bus services: $L_1^b = 2, \forall b \in B$;
- Size of additional bus fleet: $L_2 = 30$;
- Additional number of buses incurred by the introduction of localized metro-bus integration: $q_p^b = 1, \forall p \in P_b, \forall b \in B$;
- Bus service capacity: $c_b = 200$ pax/hour.

The proposed metro network resilience optimization model is coded in C++ and solved by CPLEX 12.5, and the computational experiments were run on a 3.4 GHz Core i7 PC with 16 GB of RAM. The generation of alternative paths for all OD pairs takes several minutes in total, and the two-stage stochastic model can be solved within a few seconds.

### 4.2. Metro network resilience improvement

To assess the metro network resilience improvement from localized integration with bus services, we compare the expected travel demand fulfillment under three circumstances: (1) degraded metro system without any bus service (as its intrinsic resilience); (2) degraded metro system with the existing bus services; and (3) degraded metro system with optimized bus services. The size of additional bus fleet is set as 30, and the OD data during 10am to 11am is used as the travel demand. Fig. 5 shows the metro network resilience improvement for the tested five disruption scales. As can be seen, the degraded metro system's resilience index decreases significantly with the disruption scale: the intrinsic resilience of the metro network drops from 0.90 under the least affected scale E to 0.64 under the largest magnitude A. However, the existing and optimized bus services contribute to the metro network resilience to a large extent, especially under major metro disruptive situations. Take the disruption scale A as an example, the resilience index of the degraded metro system is 0.64, while the existing bus services contribute additional 0.19. The introduction of localized integration between metro and bus systems further increases the resilience index by 0.11, which is about 58% of the resilience contribution of existing bus services. To summarize, it is feasible to utilize the existing bus services to serve those affected commuters during metro network disruptions. Besides, the connection between existing bus lines and the metro network can be further improved to enhance the service performance under metro disruptions.

### 4.3. Assessing impact of travel demand

To investigate the impact of disruptions happening during different time periods of a day on the metro network resilience improvement, we conduct computational experiments with hourly travel demands from historical smart card data. Note that pre-disruption travel demand is used as the model input, while the demand fluctuation/change in response to disruptions is not considered in this study. Given a fixed bus fleet size 30, Fig. 6 shows the metro network resilience index over the hourly travel demand under disruption scale of A. By comparing the resilience contribution from existing bus services and that from optimized bus services, we see that the resilience improvement by introducing localized integration is significant during both of peak and non-peak periods. In addition, with the optimized bus services, the travel demand fulfillment could be increased to more than 80% during non-peak hours, while more than 70% travel demand could still be satisfied during peak hours. It is worth noting that more than 90% of travel demand could be achieved before the morning peak (5 am to 7am) and in the afternoon non-peak period (10 am to 5 pm). The results also provide the expected amount of travel demand that is not satisfied under disruptions over the time periods. This could help metro operators better prepare further response actions after metro disruption happens, e.g., running bus bridging services.

### 4.4. Determining additional bus fleet size

Sensitivity analysis of the additional bus fleet size is conducted to enable metro operators to find a satisfactory trade-off between travel demand fulfillment and number of buses to be further deployed. Fig. 7 shows the sensitivity analysis of metro resilience improvement given different sizes of additional bus fleet and three time-period travel demands. As can be seen,

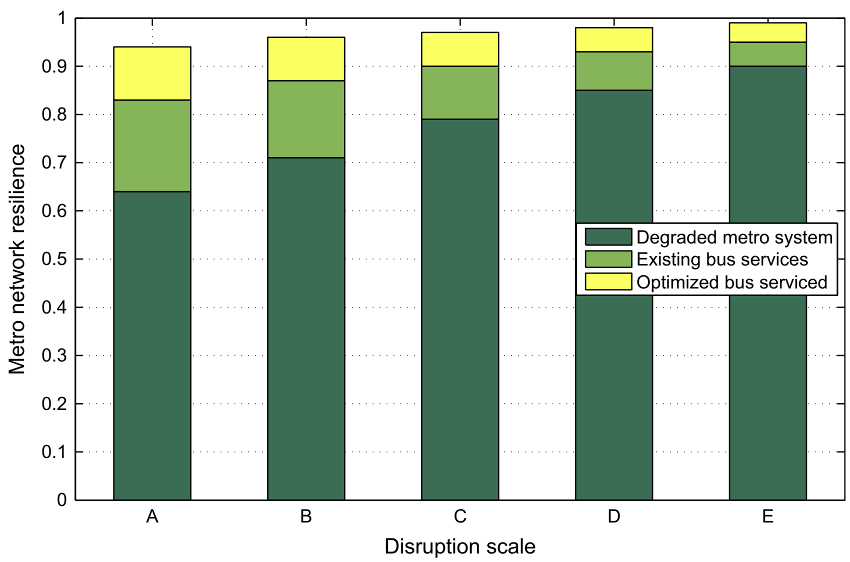


**Fig. 5.** Metro resilience improvement by localized integration with bus services.

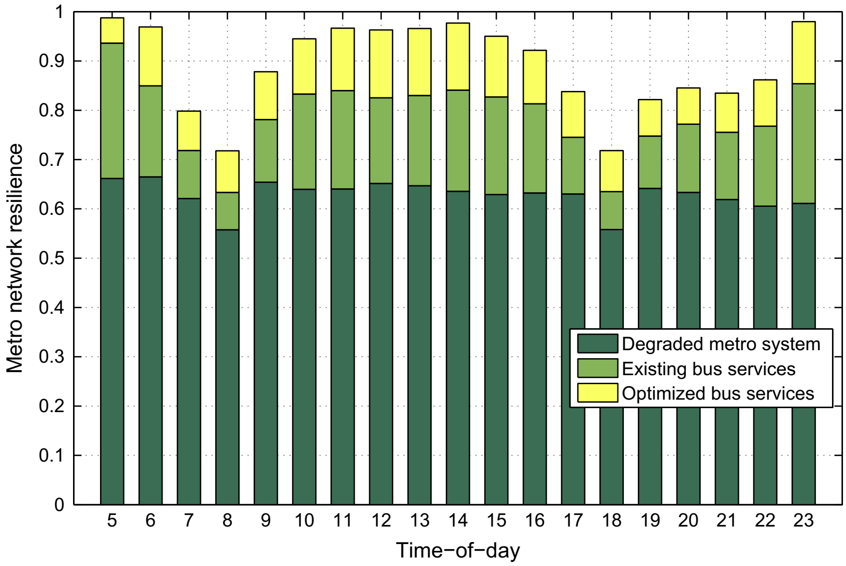


**Fig. 6.** Metro network resilience with hourly historical travel demand of an entire day.

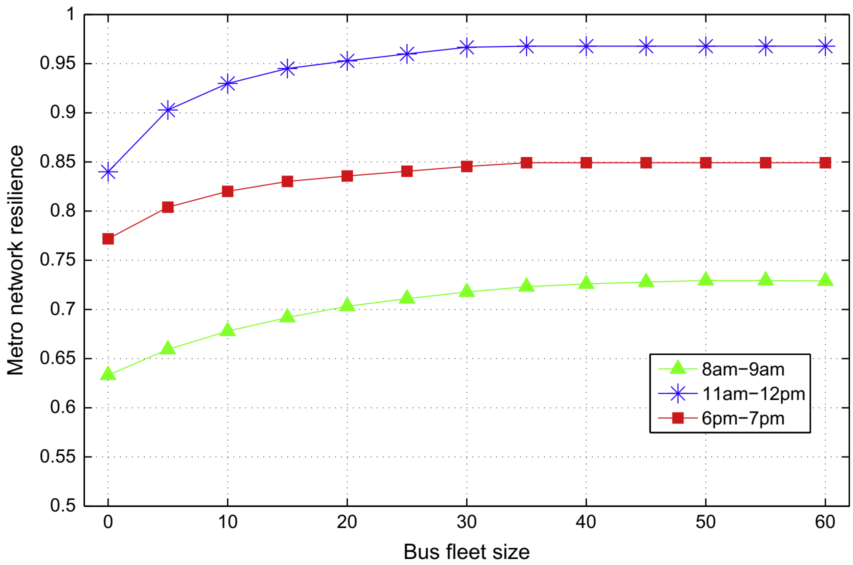


**Fig. 7.** Metro network resilience with different sizes of additional bus fleet.

the metro network resilience improves with the increase of the size of additional bus fleet. However, most of the resilience improvement comes from the firstly employed bus fleet (roughly 30 buses), while extra buses contributes to the resilience marginally.

### *4.5. Effectiveness of parallel and inter-line bus services*

Finally, we conduct computational experiments to assess the resilience improvement from different types of bus services: parallel and inter-line bus services. Fig. 8 compares the resilience indexes under four time periods and four circumstances: (1) degraded metro system without any bus services; (2) degraded metro system plus parallel bus services only; (3) degraded metro system plus inter-line bus services only; and (4) degraded metro system plus all bus services. As can be seen, both parallel and inter-line bus services contribute to the improvement of metro system's resilience, while maximum resilience improvement can be achieved by employing both of the two types of bus services.

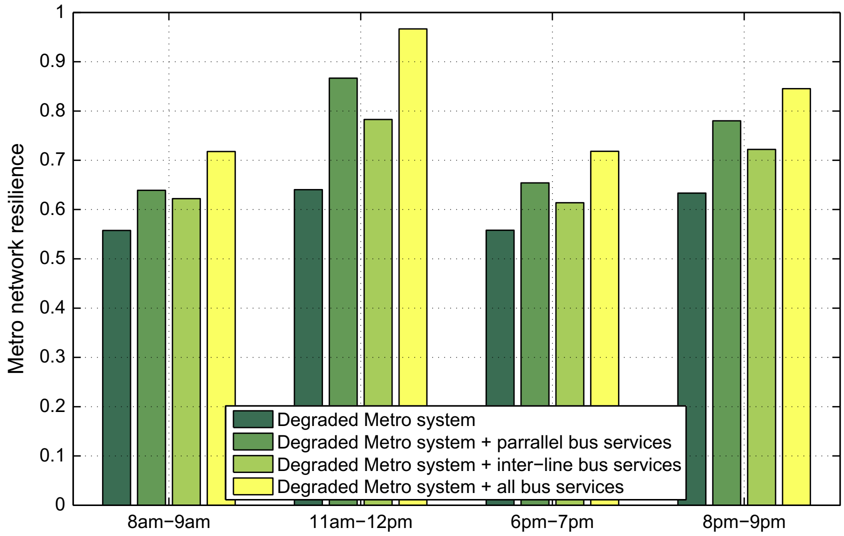


**Fig. 8.** Metro network resilience improvement from different types of bus services.

Based on the above results of the case study, we can identify the following real-life insights. (1) Despite the widely acknowledged fact that bus lines running in parallel to the metro system are not cost effective, the parallel bus services could serve as a substitute for the metro system during disruptions and fulfill, at least partially, those affected travel demand. (2) Connecting different metro lines, inter-line bus services could transfer swiftly those affected commuters from disrupted metro lines to other functioning lines during disruptions. (3) Well-connected metro-bus system guarantees superior system resilience, and require less recovery efforts during disruptions. (4) Parallel and inter-line bus services should be put in place in those areas with high probability of disruption occurrences.

## 5. Conclusion

This paper addresses the resilience enhancement for metro networks by leveraging on public bus services. The contributions of the paper to the literature include the followings:

1. A quantitative assessment for the metro system network resilience is proposed by applying Chen and Miller-Hooks (2012)'s resilience measure for intermodal freight transport networks to passenger transit networks. The resilience of metro networks is defined as the fraction of pre-disruption travel demand that can be satisfied by the degraded metro network together with the complementary bus services under disruptive conditions.
2. We adopt a proactive management strategy from the integrated metro-bus system's perspective and enhance metro system's resilience by leveraging on localized integration with bus services. An original two-stage stochastic programming model is proposed to find the best localized integration adjustments between metro and bus systems given limited bus resources and other operational restrictions. The proposed modeling framework is also capable of assessing the intrinsic resilience of metro networks, as well as the complementary capacity of existing bus services.
3. The proposed resilience optimization procedure has been tested using the data of the Singapore public transit system. The results obtained for the case study suggest the following practical observations: (a) the localized integration of metro network with bus services can enhance significantly the performance of public transport service during metro network disruptions, compared with the demand fulfillment by just the degraded metro system; (b) the obtained metro system's resilience index with different travel demand over the time-of-day provides the percentage of non-served commuters, and thus helps metro operators prepare response actions (e.g., running bus bridging services) after disruption happens; (c) localized integration between metro and bus systems should focus on those parallel and inter-line bus services which are shown to contribute to the resilience improvement of metro networks.

The most obvious practical difficulty in implementing our approach is the bus availability issue for metro system operators since it is often the case that metro and bus systems are operated by different companies. (Note that it is not an issue for Singapore as the metro companies operate both of metro and bus systems.) To solving this problem, the transportation authority could take the lead and make the cooperation between metro and bus companies possible.

For future research, we are interested in extending the developed model to integrate both pre-disruption preparedness and post-disruption recovery actions (e.g., running bus bridging service) in order to obtain an optimal resource allocation between pre- and post-disruption actions. Another promising research topic is to design the bus service network based on the given metro system from the strategic planning perspective in order to facilitate the cooperation of bus and metro companies.

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