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Demand-driven timetable design for metro services

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ABSTRACT

Timetable design is crucial to the metro service reliability. A straightforward and commonly adopted strategy in daily operation is a peak/off-peak-based schedule. However, such a strategy may fail to meet dynamic temporal passenger demand, resulting in long passenger waiting time at platforms and over-crowding in trains. Thanks to the emergence of smart card-based automated fare collection systems, we can now better quantify spatial–temporal demand on a microscopic level. In this paper, we formulate three optimization models to design demand-sensitive timetables by demonstrating train operation using equivalent time (interval). The first model aims at making the timetable more dynamic; the second model is an extension allowing for capacity constraints. The third model aims at designing a capacitated demand-sensitive peak/off-peak timetable. We assessed the performance of these three models and conducted sensitivity analyzes on different parameters on a metro line in Singapore, finding that dynamical timetable built with capacity constraints is most advantageous. Finally, we conclude our study and discuss the implications of the three models: the capacitated model provides a timetable which shows best performance under fixed capacity constraints, while the uncapacitated model may offer optimal temporal train configuration. Although we imposed capacity constraints when designing the optimal peak/off-peak timetable, its performance is not as good as models with dynamical headways. However, it shows advantages such as being easier to operate and more understandable to the passengers.

1. Introduction

With the increasing amount and range of urban mobility, making public transit more efficient has become a primary task for many cities. Owing to its greater capacity, higher speed and increased reliability, rail-based metro systems, such as Singapore's "Mass Rapid Transit" (MRT), the London Underground and the Tokyo Metro are particularly important to a metropolis. To improve service quality and reduce passenger waiting time, recent studies demonstrate an increasing interest in designing efficient operation strategies—such as adjusting train speed dynamically, increasing or decreasing the dwell time at stations and designing new service timetables—to improve metro service reliability. Of all these approaches, timetable design has been accepted as the most straightforward and effective solution.

To provide user-centric public transit services, the principle of timetable design is to meet passenger demand, reduce passenger waiting time and avoid overcrowding as far as possible (Ceder and Wilson, 1986). Without an in-depth under-

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standing of temporal demand patterns, operators' straightforward and commonly adopted strategy is a peak/off-peak-based schedule, where two types of service frequencies are set for peak and off-peak time, respectively. For example, the MRT system in Singapore operates at double frequency during morning and evening peaks. This strategy is easy to apply and performs well when supply is sufficient; however, given that passenger demand is not steady even during off-peak time, waiting time under fixed headway is still unbalanced when passenger arrival rate varies with time. On the other hand, under congested conditions or strong temporal demand heterogeneity, passengers might be unable to board a full train and they have to wait for another train. In these cases, the peak/off-peak timetable may fail to meet the temporal demand with limited supply. Furthermore, ignoring such demand dynamics may result in minor disruptions and poor service reliability. Thus, understanding the temporal demand variation and adjusting service frequency dynamically become crucial, since commuters are sensitive to their daily travel itineraries even at the minute level. Instead of assuming passengers will adjust their behavior to the service given, transit operators must better understand the nature of demand dynamics and design demand-sensitive timetables to meet greater demand with capacity-limited train services (Ceder, 2007; Niu and Zhou, 2013), reducing the risk of disruptions and attracting more ridership (Jin et al., 2013; Jin et al., 2014).

Lacking detailed passenger demand data, previous research works on scheduling mainly focused on an idealized transit system (de Palma and Lindsey, 2001; Newell, 1971; Osuna and Newell, 1972), while only a few noted the importance of passenger demand dynamics (Ceder, 1984; Ceder, 1986; Ceder, 2001). Thanks to the emergence of smart card-based automated fare collection (AFC) systems (normally on-board registration for bus systems, while off-board registration at fare gantries for metro services), we can now extract the spatial–temporally stamped journey information on an individual level from days to weeks, helping us to better understand demand variations.

In this paper, we present three optimization models for demand-driven timetable design. The first does not take train capacity as a constraint, while the second and the third allow for limited capacity constraints. The contributions of this paper are twofold; first, we propose three demand-driven timetable design models, the results of which can be further used as operational guidelines and benchmarks; second, we use smart card data to conduct an in-depth analysis of daily transit demand pattern variation and obtain detailed spatial–temporal service loading profiles.

The remainder of this paper is organized as follows: in the next section, we review previous studies on timetable design problems and the use of smart card data in transport modeling; in Section 3, we introduce our single-track timetable design problem and set out three different formulations; in Section 4, we employ a metro line in Singapore as a case and analyze the corresponding demand variation. Using the performance of the proposed models, we demonstrate the balance of different parameters against timetables in Section 5; finally, we summarize our main findings and discuss future work in Section 6.

2. Background

Previous studies on timetable design can be divided into two categories:

(1) Network timetable design problem; and
(2) Single track timetable design problem.

A network timetable design problem attempts to set up a timetable for multiple services in a connected transit network. The most common objective is to minimize transfer cost through transit coordination and synchronization among different routes. These problems have been investigated extensively on both bus networks (Ceder, 2001; de Palma and Lindsey, 2001; Ibarra-Rojas and Rios-Solis, 2012; Rapp and Gehner, 1976; Ting and Schonfeld, 2005) and metro networks (Caprara et al., 2002; Liebchen, 2008; Wong et al., 2008). Essentially, the motivation for synchronization arises from services with longer headway and higher reliability; transit services in rural areas might be a good example illustrating these characteristics. In this case, passenger waiting time can be reduced by increasing the simultaneous vehicle arrivals at transfer points. In other words, when timetables are independent (without synchronization), missing a connection will result in long delay.

In this paper, we treat the demand-driven timetable design as a single track scheduling problem. This issue was first introduced to urban bus systems as setting service frequency. Furth and Wilson (1981) proposed a model to find optimal headway by maximizing social welfare, which included both ridership benefit and waiting-time savings. The corresponding constraints were total subsidy, fleet size and occupancy levels. However, due to lack of data, the importance of time-dependent demand was not addressed and frequencies were assumed to be constants during peak and off-peak time, respectively. To characterize the time-dependent nature of transit demand and take crowding cost into account, Koutsopoulos et al. (1985) extended this model using a non-linear programming model. With the development of data collection techniques, Ceder (1984) first addressed the importance of ridership information and stated that service frequency should correspond to temporal passenger demand. This study summarized and analyzed four data collection techniques and evaluated their efficiency, using a frequency-setting problem, suggesting that a comprehensive load profile (ride check) is always superior to stop check (point check). However, the cost of such a ride check is much higher setting service because of the labor-intensive manual data collection. In a follow-up study, Ceder (1986) introduced an alternative approach for timetable design, with the objective of maximizing the correspondence of vehicle departure times with dynamic passenger demand. To evaluate the contribution of automated data collection system (ADCS) techniques, an automated procedure for efficiently setting bus time timetables was demonstrated. Given the nature of time-varying demand, Ceder (2001) proposed a scheduling model to replace constant headway by making transit vehicles even-loaded.

Compared to bus systems, a metro system is more reliable regarding operation speed, dwell time at stations and service regularity, providing a better field to simplify service-scheduling problems. Chang and Chung (2005) considered both flexibility of service rescheduling and the process of defining timetables, providing a quick response in service regulation and constructing new timetable when an incident occurs. To control irregularity caused by stochastic variations in passenger demand and traffic conditions, a real-time control strategy was introduced to maintain headway regularity by minimizing total headway variance Ding and Chien (2001). This strategy was further tested by simulating a LRT (Light Rapid Transit) service in Newark, New Jersey. The vehicle holding problem was formulated as a deterministic quadratic program for a loop network with equal scheduled headways (Eberlein et al., 2001). Peeters and Kroon (2001) proposed a method to design an optimal cyclic passenger-rail timetable, in which trains depart at the same minute every hour. By applying the periodic event-scheduling problem in a graph model (Serafini and Ukovich, 1989), Liebchen (2008) designed a timetable with shorter passenger waiting time. This timetable has been implemented on the Berlin subway system in daily operation and it is reported that both passengers and transit operators profit from this timetable. Kaspi and Raviv (2012) proposed an integrated line planning and timetabling framework with the objective of minimizing operation cost and passenger cost, which includes waiting time at both origins and transfer stations. However, for a single track timetabling problem on frequent metro services with high demand—in particular during peak hours—the primary objectives are to meet the high demand by train services with limited capacity and to reduce passenger waiting time as much as possible (Niu and Zhou, 2013). In this situation, ensuring that passengers can board a train becomes operators' primary task and understanding the variation of passenger demand becomes crucial.

The implementation of smart card-based AFC system has generated large quantities of high-quality data, recording passenger activities with detailed time and location information. It offers a comprehensive profile on passenger demand variation. It has been publicly recognized that the potential benefit of smart card data on improving public transit planning and operation is enormous (Pelletier et al., 2011). In fact, AFC can replace automated passenger counting (APC) system by recording both passenger tapping-ins and tapping-outs, helping us obtain accurate load profiles for both bus service (Lee et al., 2012) and metro service (Sun et al., 2012). Smart card data offers us an excellent opportunity to identify the demand pattern for both spatial and temporal variation.

3. Timetable design problem

In this section, we first present a detailed description of the demand-driven timetable design problem for single track metro services. Assumptions are proposed based on operational characteristics. Then, we demonstrate three mathematical programming models to design demand sensitive timetables:

- Model (A): trains without capacity constraints (optimal design).
- Model (B): trains with limited capacity (optimal operation).
- Model (C): trains with limited capacity (optimal peak/off-peak).

3.1. Modeling framework

To explore temporal variation patterns of passenger demand, we first measure the rate of metro trips on a typical weekday. Fig. 1 shows the demand variation of the whole EW line in Singapore from April 11th, 2011 to April 15th, 2011 (including both directions; averaged across weekdays), together with the temporal service headway extracted from *Google Maps*. In contrast to the pre-defined constant headway in peak and off-peak time, we see that passenger demand exhibits a significant degree of temporal variation. In this case, the metro system may suffer from greater total waiting time resulting from unbalanced passenger demand and fixed service headway. When demand is further increased, some passengers may be left behind if a coming train is full. Therefore, an optimal timetable should better meet dynamic passenger demand and reduce total waiting time, particularly when supply is limited. Our motivation is to make service timetables consistent with time-dependent passenger demand; our approach is to determine service departure times dynamically to minimize total passenger waiting time.

In a previous study, Sun et al. (2012) proposed a regression model to extract train trajectories as a first step in describing demand patterns and modeling of service operations. The result also demonstrates service reliability of the EW line of Singapore's MRT system; the extracted train trajectories are parallel with each other. Therefore, in the context of reliable operations (trains operated at same speed and without disruption), the timetable design problem is to plan train departure time from its terminus. Next, we use Fig. 2 to simplify train operation processes for convenience of formulation.

When train services are operated with high reliability, headway between two successive trains should be steady from the departure terminal to the final destination, resulting in identical trajectories across all runs (Sun et al., 2012). In this case, train trajectories can be approximated by a straight line in a spatial–temporal panel (see Fig. 2(a)). Although trains depart in continuous temporal scale, we model train departure times as discrete values to simplify this problem. In fact, formulation with discrete decision variables is more applicable and convenient for real-world operation as well. For example, the so-called "clock-face" headways are easy to memorize and operate (Ceder, 2007):

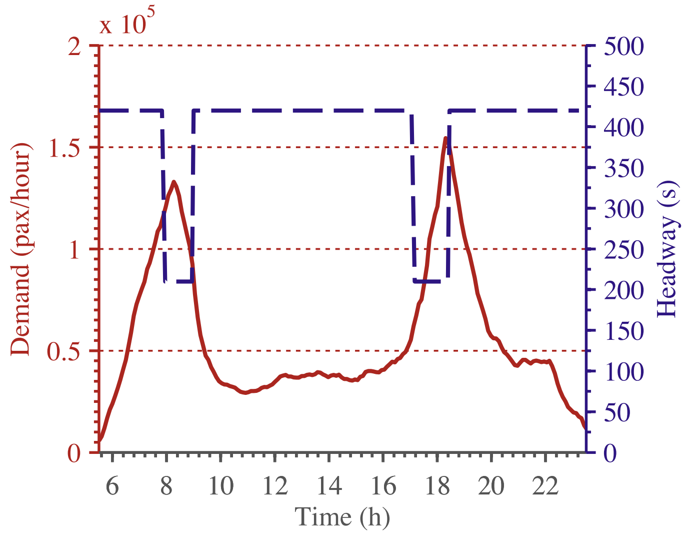

Fig. 1. Demand and headway variation of the EW line over time of day.

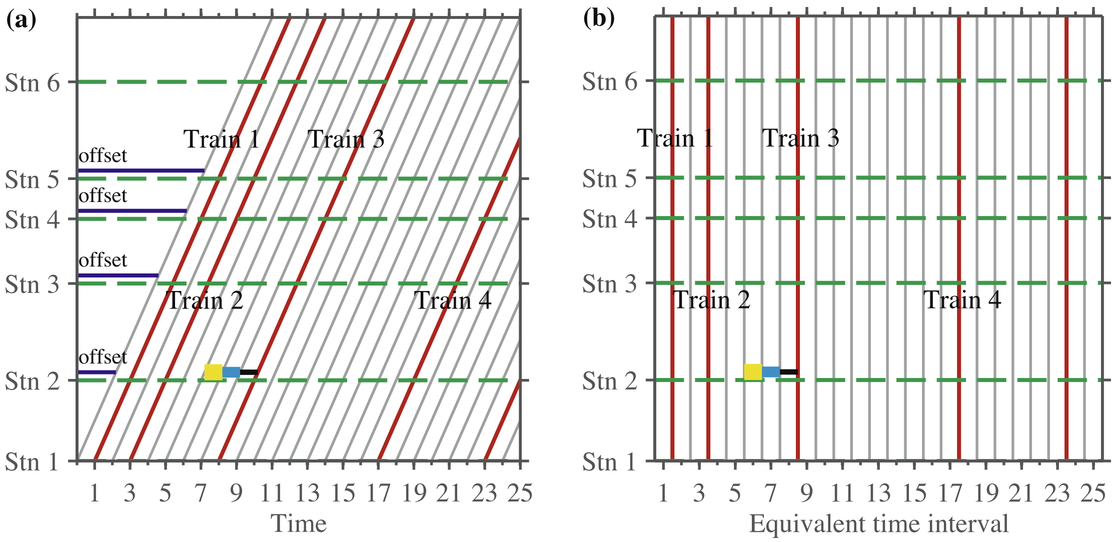

Fig. 2. Descriptions of discrete departure time and equivalent time interval.

- 6 min interval: vehicles depart at 0, 6, 12, 18, 24, 30, ..., 60 min in an hour.
- 10 min interval: vehicles depart at 0, 10, 20, 30, 40, ..., 60 min in an hour.
- 15 min interval: vehicles depart at 0, 15, 30, 45, 60 min in an hour.

Therefore, continuous time can be rescaled to discrete values given a pre-defined interval. In cases where headways are even shorter and vary over time, the interval can be defined as the greatest common divisor of headways in a similar way. Hence, time can be rescaled to discrete values as well. For example in Fig. 2(a), trains depart from the terminus at the time $t = \{1, 3, 8, 17, \ldots\}$. However, the unit of time interval can be any values, such as 30 s, 1 min, or 2 min, depending on operating requirement.

Thus, passengers at the terminus can also be grouped into time intervals given their entering times. However, considering that it takes time for trains to travel from one station to another (the operation offsets shown in Fig. 2(a)), passengers entering different stations at the same time do not share the same attribute; they may not board the same train. To better model passenger demand based on train operation, we need to rescale time correspondingly for each station. Inspired by the concept of 'moving time coordinate' (Newell, 1993), we propose the concept of equivalent time to synchronize train operation and passenger demand collectively. For any station s_i, its equivalent time is defined as the exact time minus its operation offset (see Fig. 2(b)). Through this transformation, we can re-define the passenger demand as a two-dimensional matrix—B_s^u—representing the number of passengers entering station s at equivalent time interval u.

Thus, train service operation can be simplified into a discrete process. Taking Fig. 2(b) as an example, passengers in B_2^6, B_2^7, and B_2^8 may board Train 3, which runs at the end of equivalent interval 8.

3.2. Assumptions and operational constraints

Given the previous problem description, we make the following assumptions:

3.2.1. Assumptions

A1 Reliable services: Trains run at the same speed and dwell time, their trajectories are identical. This is a major assumption to simplify service operation processes explained in the previous section. It is also a strong one, since train services are not always punctual to timetables given various disturbances. For the case in Singapore, metro systems are automatically or semi-automatically operated, making themselves more resilient to disturbance by adjusting speed and dwell time in real-time. This assumption was tested in a previous study (Sun et al., 2012), where we find that trains can be operated regularly in disruption-free scenarios from the observed identical trajectories.

A2 Uniform arrival: For any station s, passengers arriving at equivalent interval t are distributed uniformly, which is proposed to simplify measurement of total waiting time. A number of service reliability studies suggest that this assumption is reasonable for transit service with short headway (such as metro services). However, passengers may have prior knowledge about service timetables when frequency is low and they tend to adjust their departure time accordingly (Furth and Muller, 2006). The operation constraints are identified before we formulate the model. We introduce the following operation constraints:

3.2.2. Constraints

C1 Discrete departure time: trains are restricted to depart at the end of equivalent interval t.
C2 Operation cost: number of daily services is fixed.
C3 Operation safety: headway should not be less than the minimum requirement.
C4 Service level: all passengers should be accommodated within a certain time, such as 20 min.
C5 Last service: departure time of the last service is predefined.

3.3. Model formulation

We now formulate mathematical programming problems that determine the departure time of each service. To measure passenger waiting time, we first define the set of waiting profiles p based on equivalent time intervals:

p: if passengers who enter stations in interval u board the train service departing at the end of interval $t(t \geqslant u)$, their waiting profile is defined as $p = t - u + 1$ (see Fig. 3 for description).

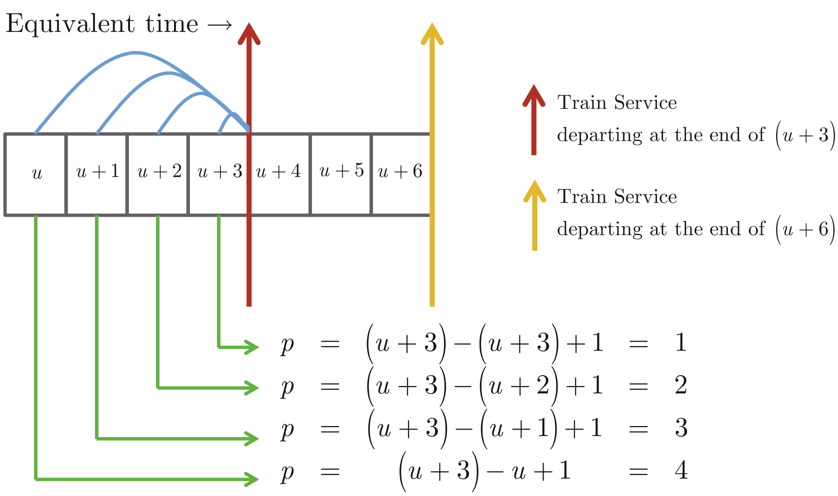

Fig. 3. Description of waiting profile p.

Given the uniform arrival assumption, passengers have an average wait time $p - 0.5$ (in number of equivalent intervals) if they choose waiting profile p. Assuming the designed timetable should serve every passenger in time, i.e. Constraint C4, the set of waiting profile should be upper bounded. Taking all assumptions and constraints, we formulate the timetable design process as an optimization problem minimizing passengers' total waiting time. Before presenting the formulations, we first define the following notations.

3.3.1. Notation

The following sets, indices, and parameters are used:

Sets:

T: set of equivalent time intervals. $T = \{1, 2, \ldots, T_n\}$.
S: set of stations, excluding the final destination. $S = \{1, 2, \ldots, S_n\}$.
P: set of waiting profiles. $P = \{1, 2, \ldots, P_n\}$.

Indices:

t: index of equivalent time intervals for train services. $t \in T$.
u: index of equivalent time intervals for passengers. $u \in T$.
s: index of metro stations. $s \in S$.
p: index of waiting profiles. $p \in P$.

Parameters:

T_n: number of equivalent time intervals.
S_n: number of metro stations, excluding the final destination.
K_n: number of daily train services.
P_n: number of waiting profiles.
B_s^u: demand profile, representing number of passengers arriving at station s in equivalent time interval u.
$N_{\max}$: maximum service headway, in number of equivalent intervals.
$N_{\min}$: minimum service headway, in number of equivalent intervals.

3.3.2. Model (A)

Here we assume that trains have unlimited capacity. In other words, a train can carry all passengers waiting for it. Given the reliable service assumption, as long as passengers arrive at a station in the same equivalent interval u, they will share the same waiting profile (i.e. the average waiting time of passengers groups $B_s^u\ \forall s \in S$ is the same). In this model, we define two sets of decision variables:

Decision variables:

$x_t :\in \{0, 1\},\ \forall t \in T$: 1 if a train departs from terminal at the end of equivalent interval t; and 0 otherwise.
$y_{u,p},\ \forall u \in T,\ \forall p \in P$: the proportion of passengers entering stations in equivalent interval u choosing waiting profile p.

Given the uniform arrival assumption, the average waiting time w^u of passenger entering at equivalent u can be calculated as:

$$w^u = \sum_{p \in P} y_{u,p}(p - 0.5) \tag{1}$$

By defining total passenger demand in equivalent interval u across all stations as $B^u = \sum_{s \in S} B_s^u$, the timetable design problem is formulated in the following mathematical programming:

[Model (A)]

$$\min \sum_{u \in T} B^u w^u \tag{2}$$

subject to:

$$x_t \in \{0, 1\}, \quad \forall t \in T \tag{3}$$

$$\sum_{t \in T} x_t = K_n \tag{4}$$

$$\sum_{t \in [t_1, t_2]} x_t \leqslant 1, \quad t_2 = t_1 + N_{\min} - 1,\ \forall t_1, t_2 \in T \tag{5}$$

$$\sum_{t \in [t_1, t_2]} x_t \geqslant 1, \quad t_2 = t_1 + N_{\max} - 1,\ \forall t_1, t_2 \in T \tag{6}$$

$$x_{T_n} = 1 \tag{7}$$

CA1

$$0 \leqslant y_{u,p} \leqslant 1, \quad \forall u \in T, \ \forall p \in P \tag{8}$$

CA2

$$\sum_{p \in [1, \min(T_n - u + 1, P_n)]} y_{u,p} = 1, \quad \forall u \in T \tag{9}$$

CA3

$$x_t \geqslant y_{u,p}, \quad t = u + p - 1, \ \forall t \in T, \ \forall p \in P \tag{10}$$

Objective function (1) minimizes the total waiting time over all passengers. Constraints (2)–(7) correspond to the pre-defined operational constraints C1–C5. Constraint (8) and (9) ensure that all passengers are assigned waiting profiles. $p \in [1, \min(T_n - u + 1, P_n)]$ guarantees that the size of waiting profiles is reduced for passenger arriving after $u = T_n - P_n$. Constraint (10) guarantees consistency between boarding passengers and train services; if there are passengers assumed to board at the end of interval u, there must be one train serving people at that equivalent time (see Fig. 3). Taken together, Model (A) is formulated as a mixed integer programming (MIP) problem.

A strong assumption of this model is that trains have unlimited capacity and passengers can always board the first coming train. In other words, this result provides us the optimal timetable only if service capacity is sufficient to carry all the waiting passengers. Nevertheless, if we consider the current metro operation as a feasible case with few people unable to board due to the capacity constraint, Model (A) still has the potential to design a more dynamical timetable compared to the peak/off-peak schedule. Furthermore, Model (A) may provide implications on the optimal time-dependent service capacities. By applying the optimal timetable on the current demand, we may get the maximum occupancy in temporal scale, helping us identify best train configurations over time of day. For example, in order to provide more space for standees and to increase the total capacity using the same number of train cars, operators usually run coaches with fewer seats in peaks hours in Singapore.

However, when demand further increases or temporal heterogeneity becomes more significant, Model (A) may fail to capture the additional waiting time caused by passengers left behind by a full train, particularly during peak time. To address this limitation, we extend Model (A) to take these capacity constraints into account.

3.3.3. Model (B)

To model limited service capacity and its consequences, the following additional parameters are introduced:

Parameters:

CAP: service capacity (number of onboard passengers should not exceed CAP).

$B^u_{s,d}$: temporal demand profile indexed by both boarding station and alighting station, representing number of passengers entering station s in equivalent interval u with destination d. By dividing $B^u_s = \sum_{d \in S} B^u_{s,d}$ in Model (A) using destinations, we are able to quantify the spatial occupancy of each train.

Decision variables:

$x_t :\in \{0, 1\}$: 1 if there is one train serving at the end of equivalent interval t; and 0 otherwise.

$y^s_{u,p}$: The proportion of passengers entering station s in equivalent interval u choosing waiting profile p. This variable is an extension of $y_{u,p}$ in Model (A), since passengers at different stations no longer share similar attributes. If the surplus capacity of the first coming train is limited, only part of the waiting passengers

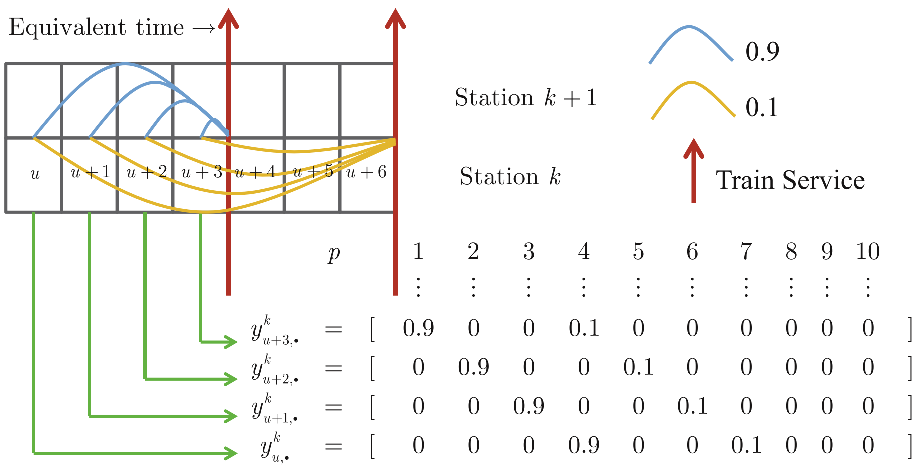

Fig. 4. Description of $y^s_{u,p}$ ($\cdot$ means all waiting profiles $p \in P$ ($P_n = 10$)).

can board the train; the rest have to wait for the following service. For the example shown in Fig. 4, 90% of the waiting passengers who entered station k at u to $u+3$, can board on the first train. Therefore, $y^s_{u,p}$ is a continuous value within $[0,1]$ in this model.

q^s_t: occupancy of train services departing at the end of t at station s ($s \in [1, S_n - 1]$) after alighting and boarding.

Because of capacity constraints, passengers arriving in u no longer share the same waiting profile, as they might be unable to board if the coming train is full. Hence, the average waiting time of passenger group B^u_s is calculated as:

$$w^u_s = \sum_{p \in P} y^s_{u,p}(p - 0.5) \tag{11}$$

Then, the capacity incorporated timetable design problem is formulated as:

[Model (B)]

$$\min \sum_{s \in S} \sum_{u \in T} B^u_s w^u_s \tag{12}$$

where $B^u_s = \sum_{d \in S} B^u_{s,d}$.

subject to:

$$x_t \in \{0, 1\}, \quad \forall t \in T \tag{13}$$

$$\sum_{t \in T} x_t = K_n \tag{14}$$

$$\sum_{t \in [t_1, t_2]} x_t \leqslant 1, \quad t_2 = t_1 + N_{\min} - 1, \ \forall t_1, t_2 \in T \tag{15}$$

$$\sum_{t \in [t_1, t_2]} x_t \geqslant 1 \quad t_2 = t_1 + N_{\max} - 1, \ \forall t_1, t_2 \in T \tag{16}$$

$$x_{T_n} = 1 \tag{17}$$

CB1

$$0 \leqslant y^s_{u,p} \leqslant 1, \quad \forall s \in S, \ \forall u \in T, \ \forall p \in P \tag{18}$$

CB2

$$\sum_{p \in [1, \min(T_n - u + 1, P_n)]} y^s_{u,p} = 1, \quad \forall s \in S, \ \forall u \in T \tag{19}$$

CB3

$$x_t \geqslant y^s_{u,p}, \quad t = u + p - 1, \forall s \in S, \forall t \in T, \forall p \in P \tag{20}$$

CB4

$$q^s_t = \begin{cases} \sum_{u \in [\max(1, t - P_n + 1), t]} \sum_{d \in [2, S_n]} B^u_{1,d} y^1_{u,t-u+1} & s = 1, \ \forall t \in T \\ q^{s-1}_t - \sum_{u \in [\max(1, t - P_n + 1), t]} \sum_{o \in [1, s-1]} B^u_{o,s} y^o_{u,t-u+1} + \sum_{u \in [\max(1, t - P_n + 1), t]} \sum_{d \in [s+1, S_n]} B^u_{s,d} y^s_{u,t-u+1} & \forall s - 1, \ s \in S, \ \forall t \in T \end{cases} \tag{21}$$

CB5

$$q^s_t \leqslant CAP, \quad \forall s \in S, \forall t \in T \tag{22}$$

Objective function (23) minimizes the total waiting time. Constraints (13)–(17) correspond to the pre-defined operational constraints. Additional constraints CB1 $\sim$ CB5 are due to the following:

Constraint (18) and (19) ensure that all passengers are assigned waiting profiles. Constraint (20) guarantees consistency between boarding passengers and train services as well; if there are passengers supposed to board at the end of interval u, there must be one train serving people at that equivalent time (see Fig. 4). Constraints (21) quantifies the spatial–temporal occupancy. Constraint (22) corresponds to the limited capacity, ensuring that occupancy is not more than capacity. Similar to Model (A), Model (B) is also a MIP; however, the problem is much larger, considering that the decision variable $y_{u,p}$ has been increased for one dimension to $y^s_{u,p}$ and additional constraints regarding service capacity have been added.

3.3.4. Optimal capacitated peak/off-peak timetable

We also introduce another model to find optimal peak/off-peak timetable. We define two additional parameters:

Parameters:

N_{Peak}: headway (in number of intervals) in peak periods.

N_{Off}: headway (in number of intervals) in off-peak periods. To avoid too much imbalance between peak and off-peak services. We let $N_{Off} < 3N_{Peak}$.

This model has the same decision variables and objective function as Model (B).

[Model (C)]

$$\min \sum_{s\in S}\sum_{u\in T} B_s^u w_s^u \tag{23}$$

subject to Constraints (13), (14), (17)–(22) and:

$$x_{t+N_{Peak}} + x_{t+N_{Off}} \geqslant x_t, \ \forall t, t+N_{Peak}, t+N_{Off} \in T \tag{24}$$

$$\sum_{t\in[t_1,t_2]} x_t \leqslant 1, \quad t_2 = t_1 + N_{Peak} - 1, \ \forall t_1, t_2 \in T \tag{25}$$

Constraint (24) ensures that headway should be chosen as N_{Off} of N_{Peak}. On the other hand, we need to also avoid both peak and off-peak headways being selected at the same time. In doing so, we can simply impose another constraint:

$$x_{t+N_{Peak}} + x_{t+N_{Off}} \leqslant 1, \ \forall t, t+N_{Peak}, t+N_{Off} \in T \tag{26}$$

when $N_{Off} \neq 2N_{Peak}$. If $N_{Off} = 2N_{Peak}$, Constraint (26) should be removed as both $x_{t+N_{Peak}}$ and $x_{t+N_{Off}}$ can be 1 during peak period.

Note that this model does not limit the number of transitions between peak and off-peak strategy. However, given the strong temporal demand pattern, we do expect the model to provide timetables with a consistent peak/off-peak structure.

3.4. Complexity analysis

The metro service timetable design problem without capacity consideration is equivalent to the one-dimensional *Facility Location Problem* (Love, 1976) where all facilities and customers are all restricted to a Euclidean line: trains and passengers correspond to facilities and customers, respectively; the train departure decision corresponds to the facility location selection; the passengers' waiting time corresponds to the distance from demand points to facility locations; the amount of passengers corresponds to the weighting factor of demands. The unique feature of the metro timetable design problem is that demands can be only assigned to facilities with larger coordinates along the line since passengers can only board trains after their arrival. To solve the uncapacitated version of the location problem on a line, Love (1976) proposed a dynamic programming algorithm, which can be easily adopted to solve the uncapacitated metro service timetable design problem with minor adjustment. Thus, the uncapacitated metro service timetable design problem can be solved in polynomial time.

Similarly, if we consider a special case of the capacitated metro service timetable design problem where the minimum or maximum headway consideration is not considered, the resulted problem is equivalent to the capacitated *p-Facility Location Problem* on a real line (Brimberg et al., 2001). The authors proved that the addition of capacity constraints for facilities renders the problem to be NP-hard. Thus, the train service timetable design problem with capacity consideration is also NP-hard.

In this paper, we simply employ standard solvers (e.g., CPLEX) to solve the uncapacitated and capacitated metro service timetable design problems (Model (A), (B) and (C)), as computational experiments show that both models can be solved within acceptable time.

4. Case study

So far, we have formulated three different MIPs on the single-track timetable design problem. To test performance of the proposed timetable design formulations, we conduct a case study based on one MRT line in Singapore. In this section, we first introduce the smart card transaction data and then briefly discuss the case line.

4.1. Data preparation

Since the late 1990s, smart card-based automated fare collection (AFC) has been widely applied in cities all over the world, such as London (Oyster), Tokyo (Suica), Singapore (CEPAS) and Washington (Smarttrip). The AFC system was introduced in Singapore in April, 2002. With increasing usage over the last decade, it now covers all metro trips and more than 93% of bus trips (the rest are cash payments). Although the goal of this system is to make fare collection faster and more convenient for transit users, the system also generates large quantities of data, recording time and location-stamped transit use for each individual passenger.

Compared to entry-control system, such as in Outaouais Region, Canada (Trépanier et al., 2007) and Santiago, Chile (Munizaga and Palma, 2012), this AFC system in Singapore records both entry and exit information, making the data source highly comprehensive for research purposes. To better understand the demand variation of metro services, we employ one week's smart card transactions from April 11th, 2011 to April 17th, 2011. The data recorded both bus and MRT trips. A full trip record can be represented as a tuple (id, t_o, l_o, t_d, l_d), suggesting that passenger *id* departed from origin station/stop l_o at the time t_o and arrived at destination station/stop l_d at the time t_d. This information enables us to obtain the input parameters $B_{s,d}^u$ for the timetable design problem.

Table 1
Time offset table for estimating demand over equivalent time.

Station	1	2	3	4	5	6	7	8	9	10	11	12
Offset [s]	0	178	309	492	646	806	922	1039	1160	1291	1408	1525
Station	13	14	15	16	17	18	19	20	21	22	23	...
Offset [s]	1637	1749	1870	1982	2118	2239	2370	2491	2608	2739	2884	...

4.2. Case: the EW Line

We chose one line from the MRT system in Singapore as an example. To convert real time to the defined equivalent time interval, we applied the travel time regression model in (Sun et al., 2012) to estimate the offset for each station. The results are provided in Table 1.

4.3. Passenger demand

Smart card data has been used to study transit macroscopic demand patterns and their variation by many researchers. For example, Agard et al. (2006) analyzed smart card data collected from Outaouais Region, Canada and identified different trip habits and variability based on pre-defined user types. Utsunomiya et al. (2006) found that the demand pattern varied with day of the week, especially for weekdays and weekends. Therefore, transit operators are urged to use different schedules from day to day. Park et al. (2008) investigated and characterized the demand pattern of different transport modes in Seoul, South Korea. However, we need to measure the temporal demand $B^u_{s,d}$ in microscopic detail. In order do so and characterize the variability of B^u_s from day to day, we first distinguish two types of metro trips:

Full trips: Both entry and exit stations are on the selected service line.
Partial trips: Only part of the metro trip is on the selected line. (At least one station is not on the line.)

Therefore, the demand matrix B^u_s is the combination of full and partial trips. As opposed to bus systems, the tapping-in time is registered at fare gantries instead of on transit vehicles, making it easy for us to obtain the temporal demand. However, for partial trips, we have to identify which segments of the selected line the passenger has traveled, and then add this part on to the full trip demand.

To solve this problem, we apply the MATSim agent-based transport simulation toolkits to reconstruct the daily scenario (MATSim, 2013). To find the segment these passengers traveled (s and d on the subject line), partial trips are simulated on the whole metro network. Using travel time as utility, we find that about half of the trips on the selected line are full trips and the other half are partial trips. After combining these two demand sources, we begin to estimate the final temporal demand B^u_s.

Using the calculated time offsets and directed/transfer trip information, we estimate each element in the temporal demand B^u_s. Fig. 5 plots the average demand profile for different stations over weekdays. As can be seen, demand is smooth and steady for most times of day; however, significant morning and evening peaks emerge at most stations.

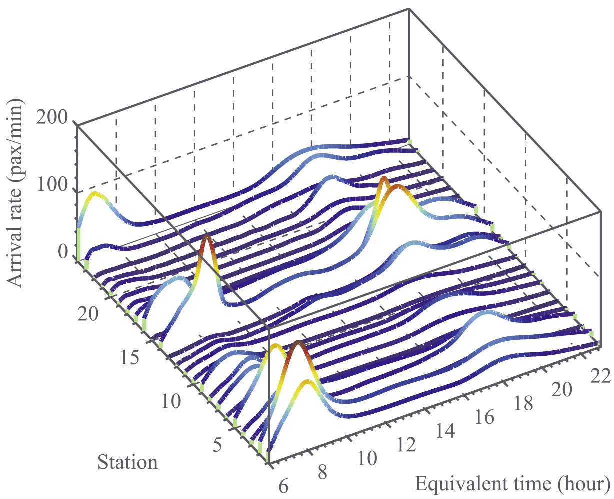

Fig. 5. Passenger demand over equivalent time.

Table 2
Cosine similarities of boarding demand and of on-board demand.

Sim_B	(Mon)	(Tue)	(Wed)	(Thu)	(Fri)	(Sat)	(Sun)
(Mon)	–	.97	.97	.96	.96	.76	.68
(Tue)	.97	–	.97	.96	.96	.76	.68
(Wed)	.97	.97	–	.97	.96	.76	.69
(Thu)	.96	.96	.97	–	.96	.77	.69
(Fri)	.96	.96	.96	.97	–	.80	.73
(Sat)	.76	.76	.76	.77	.80	–	.92
(Sun)	.68	.68	.69	.69	.73	.92	–

4.4. Demand variation

To explore the day-to-day variation of passenger demand, we use cosine similarity to measure the spatial–temporal variation of both boarding demand and on-board demand. First, for boarding demand, after filling the demand matrix $B_s^u(\forall s, \forall u)$ for each day, the boarding demand similarity between day i and day j is defined as:

$$Sim_B(i,j) = \frac{\sum_{s\in S}\sum_{u\in T} b_{s,u}^i b_{s,u}^j}{\sqrt{\sum_{s\in S}\sum_{u\in T}(b_{s,u}^i)^2}\sqrt{\sum_{s\in S}\sum_{u\in T}(b_{s,u}^j)^2}} \tag{27}$$

where $b_{s,u}^i = \frac{B_s^u(i)}{\sum_s\sum_u B_s^u(i)}$ is distribution of boarding demand at day i. Therefore, the value of similarity is between 0 and 1. The more similar the two matrices are, the nearer to 1 Sim_B is.

Based on the demand data, we calculate Sim_B during weekdays. Table 2 shows the cosine similarity of both boarding demand and on-board demand with equivalent interval length $\Delta\tau = 1$ min. Note that all values of weekday boarding demand similarity are larger than 0.95, indicating a strong similarity between each day; day-to-day demand variation is tiny and negligible. The analysis shows that daily transit demand on the selected line exhibit a strong degree of homogeneity.

Hence, for convenience, we use an average demand profile over weekdays to test the proposed models' performance in the following analysis.

5. Results and analysis

In this section, we evaluate the performance of these three models based on the case line introduced in previous section. We consider two cases here: Case 1 is a major problem, covering the entire metro line across whole operation time; Case 2 is a minor problem confined to the morning peak, consisting of only 15 stations. The detailed parameters of these two cases are listed in Table 3. We first use Case 1 to compare the overall performance of Model (A), (B) and (C).

All computation experiments were conducted on a PC with an Intel Core i7 3.40 GHz with 16 GB RAM. The proposed Models (A), (B) and (C) are coded in CPLEX solver 12.5, using the standard configuration of CPLEX solver, which usually employs the Branch-and-Bound algorithm for MIP models. Table 4 summarizes the computational costs of these three models using both the major and minor cases.

5.1. Optimal results

As the length of peak period headway and off-peak headway are parameters in Model (C). Based on the constraints on headway $N_{min} \leqslant N_{Peak} < N_{Off} \leqslant N_{max}$ and $N_{Off} < 3N_{Peak}$, the parameter set of (N_{Peak}, N_{Off}) is $\{(2,3),(2,4),(2,5),(3,4),(3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8),(4,9),(4,10),\ldots\}$. We tested all possible combinations of (N_{Peak}, N_{Off}), finding that those feasible are (3,8), (4,7), (4,8), (4,9), (4,10) and (5,7). In all these cases, the model provides consistent peak/off-peak timetables with four transitions. The least waiting time is obtained when $N_{Peak} = 4$ and $N_{Off} = 8$.

Table 3
Input parameters of case studies.

Parameter	Case 1	Case 2
$\Delta\tau$	1 min	1 min
T_n	1021 (6:00–23:00)	181 (6:00–09:00)
S_n	24	15
K_n	165	40 [35–55]
P_n	20 (maximum waiting time 20 min)	20 (maximum waiting time 20 min)
N_{max}	10 (maximum headway 10 min)	10 (maximum headway 10 min)
N_{min}	2 (minimum headway 2 min)	2 (minimum headway 2 min)
B_s^u	Obtained by averaging weekday demand	Obtained by averaging weekday demand
CAP	2000 pax/service	2000 [1900–2500]

Table 4
Computational results of Model (A) and (B).

Running time [s]	Case 1 (Major)	Case 2 (Minor)
Model (A)	1.17	0.20
Model (B)	>3600.00	161.17
Model (C)	>3600.00	–

To explore the variation of optimal timetables from these models, we plot the temporal headway by calculating the difference between sequential departure times for three scenarios in Fig. 6(a): (1) peak/off-peak, (2) optimal solution of Model (C), (3) optimal solution of Model (A) and (4) optimal solution of Model (B), respectively. As can be seen in Fig. 6(a), timetable (3) and (4) display more consistence with temporal passenger demand compared to (1) and (2). The figure also helps us distinguish the optimal solutions from each other. Owing to capacity constraints, we see clearly that timetable (4) operates more trains than timetable (3) during morning peaks.

To estimate the desired occupancy (when no passengers are left behind) of train services, we map passenger demand $B^u_{s,d}$ on timetable (1) and (3), respectively, to calculate the corresponding spatial temporal occupancy of each train. We determine the maximum occupancy of a train departed at the end of t as $Q_t = \max_{s \in S} \{q^s_t\}$ and use it to measure the importance of capacity constraints for all the three timetables (Fig. 6(b)). Obviously, Q_t is always less or equal to 2000 for Model (B) and (C) (see the second and the bottom inset of Fig. 6(b)). However, we found that Q_ts of both timetable (1) and (3) exceed 2000 during morning and evening peaks, suggesting that the passenger demand at these equivalent intervals may not be met effectively if service capacity is fixed at 2000, resulting in additional waiting time when some passengers are left behind.

To measure the actual/congested waiting time W'_u of timetable (1) and (3), we calculate the spatial occupancy of each train based on $B^u_{s,d}$ by tracking parameter ρ^u_s, demonstrating the ratio of left-behind passengers. To quantify ρ^u_s, we first map the tested timetable on the same panel as in Fig. 2(b) and assume that left-behind passengers and new arriving passengers have the same chance to board on the coming train when occupancy reaches capacity. Then, the ratio ρ of left-behind passengers over all waiting passengers for each station and each train can be calculated. The estimation of ρ^u_s also helps us measure the total waiting time $W_{s,k}$ for Train k at Station s. In general, $W_{s,k}$ consists of waiting time of boarding passengers $W^1_{s,k}$ and left-behind passengers $W^2_{s,k}$. Taking Station 2 in Fig. 2(b) as an example, if no passengers are left by Train 2, passengers who arrived in equivalent interval 4–8 have the same chance to board train 3, with total waiting time measured as $W^1_{2,3} = \sum_{u=4}^{8}(1-\rho)B^u_{s,\cdot}(8-u+0.5)$, where $B^u_{s,\cdot} = \sum_{d \in S} B^u_{s,d}$. Next, we can move the passengers left behind $\rho B^u_{s,\cdot}(4 \leqslant u \leqslant 8)$ to $B^9_{s,\cdot}$ and charge them with full waiting time as: $W^1_{2,3} = \sum_{u=4}^{8} \rho B^u_{s,\cdot}(8-u+1)$. By applying this procedure iteratively, we can measure both spatial–temporal occupancy and total waiting time under capacity constraints simultaneously.

As Table 5 shows, timetable (4) performs best and average waiting time from timetable (3) is 6.2% higher. The average waiting time of the timetable (2) is about 3.23 min (10.5% higher than (4)). The current peak/off-peak timetable (from *Google Maps*) offers the longest waiting time (46.3% higher than (4)). We also measured the number of passengers that are

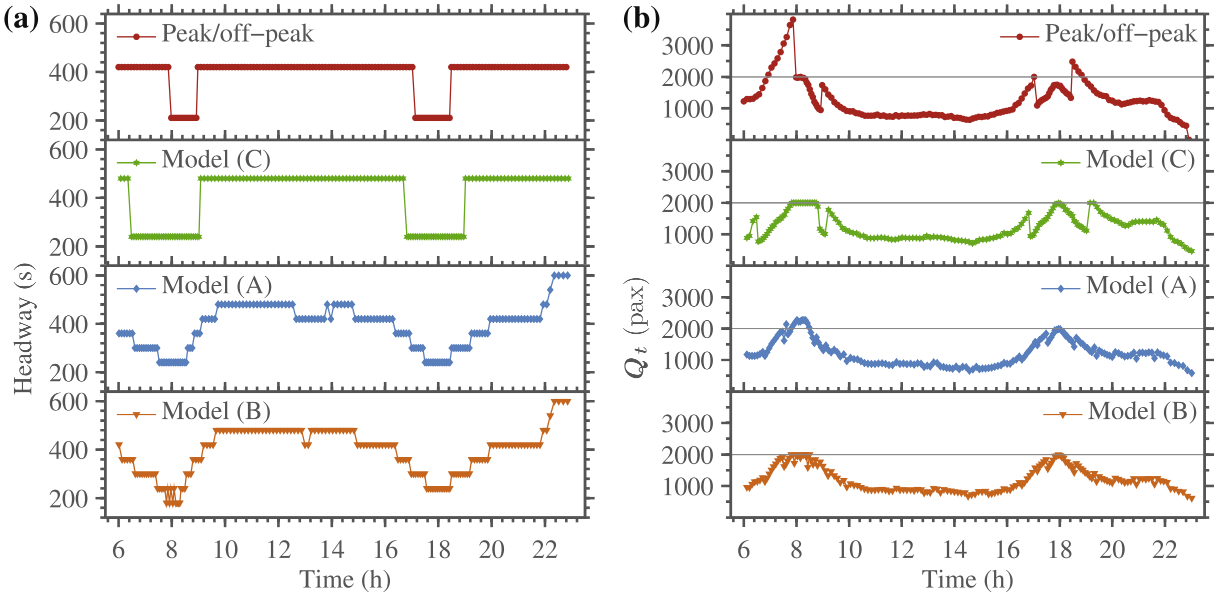

Fig. 6. Headway profile and maximum occupancy from peak-based schedule, Model (C), Model (A) and Model (B).

Table 5
Waiting time under different scenarios.

	Timetable	W'_u [min]	Left-behind (pax)
(1)	Peak/off-peak	4.277 (+46.3%)	147,434
(2)	Model (C)	3.232 (+10.5%)	26,924
(3)	Model (A)	3.106 (+6.2%)	19,991
(4)	Model (B)	2.924 (+0.0%)	638

left-behind by the first coming train. In this case, the peak/off-peak timetable is the worst because of the large number of passengers who cannot board due to limited service capacity (about 231 times larger than timetable (4)). Timetable (2) and (3) are comparable with number of left behind 26,924 (42 times) and 19,991 (31 times), respectively.

To test the robustness of our results, we conducted a training/testing experiment by using demand from Monday to Wednesday as input to determine optimal timetable (4), and testing such timetable on passenger demand on Thursday and Friday, respectively. The timetable determined by demand from Thursday is identical to the one from Monday to Wednesday. Applying the same timetable on demand of Friday results in average waiting time 2.93 min, while the waiting time of the best timetable (estimated using Fri demand) is 2.91 min. Given the result, we would argue that variation from daily operation is marginal in determining the timetable compared to weekly, monthly or seasonal variation of demand. Nevertheless, given that smart card data is available from daily operation, we suggest using demand from previous week to design timetables for the current week.

To further quantify the benefit of dynamical timetables under demand variation, we access their performance by adjusting the current demand (from 85% to 115%). Fig. 7 shows the sensitivity analysis of average waiting time and number of delayed passengers given the demand variation. As can be seen, the demand sensitive timetable (2), (3) and (4) show higher resilience to the increase in demand than the current peak/off-peak timetable. However, timetable (4) still performs good when demand increases 10%, while timetable (2) and (3) becomes more sensitive. Taken together, all the three models can provide better timetables than the current peak/off-peak timetable.

Although capacity constraints are imposed on Model (C), timetable (2) performs not as well as (3) and (4) with current demand, being limited by the strict choices of N_{Peak} and N_{Off}. On the contrary, Model (A) and (B) can still benefit from the dynamical headway. Still, the optimal peak/off-peak schedule shows some advantages such as being easier to operate and more understandable to passengers. Model (A) is as good as (B) when passengers are seldom delayed by the capacity constraints (or train capacity is large enough). However, obviously, if passenger demand is further increased, Model (B) will have significant advantages over Model (A) and (C). In fact, the solution of Model (A) gives a lower bound of average waiting time since it offers equal weight to each passengers no matter if he/she can board or not. Any factors causing the violation of capacity constraints will prevent Model (A) from performing optimally. Despite increasing passenger demand, some other factors can also result in more passengers being delayed by capacity constraints, such as:

- Reduced number of services.
- Reduced service capacity.

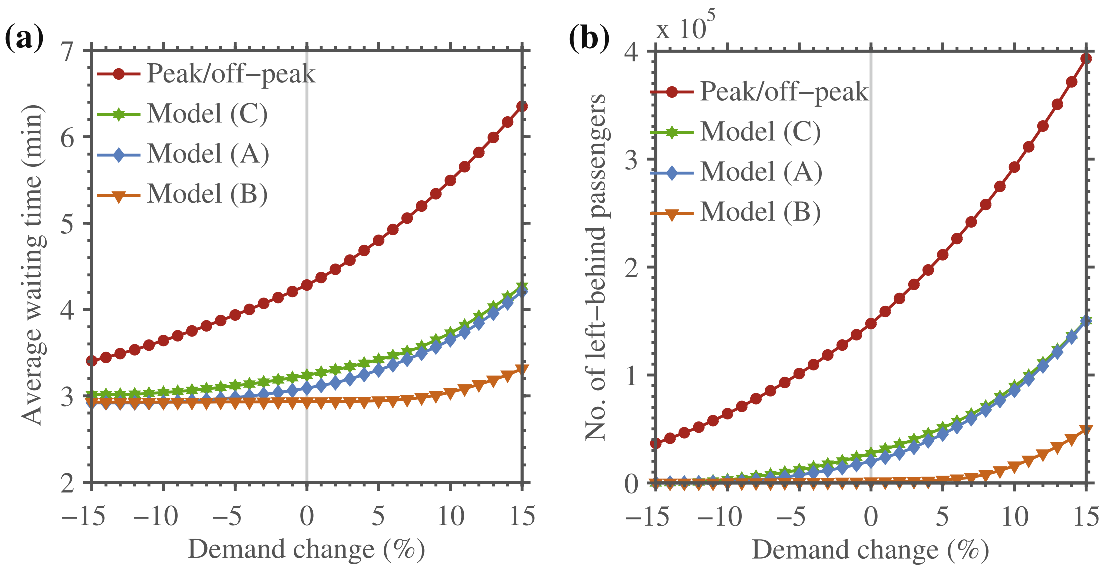

Fig. 7. Performance of timetables under demand variation.

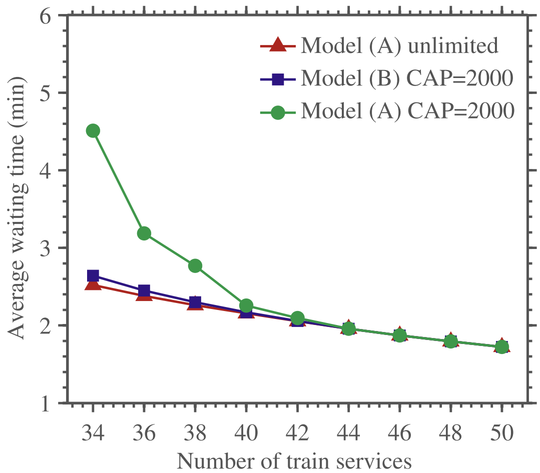

Fig. 8. Balancing number of services and timetable.

Therefore, there are trade-offs among number of services, service capacity and timetable. Next, we use Case 2 to assess these balances for Model (A) and (B).

In fact, as stated before, Model (A) provides biased results under congested condition. In this case, we cannot tell which model is good taking only the computation time into account. To further evaluate their performance, we need to analyze the results and perform sensitivity analysis.

Considering that the computation time for Model (B) is very long for the full case (Case 1), we use the minor case (Case 2) to test sensitivity on number of services K_n and service capacity *CAP*.

5.2. Balancing number of services and timetable

We use morning peak hours from 6:00 to 9:00 to test the impact of number of services—K_n. For the peak/off-peak timetable, $K_n = 36$. To analyze the impact of K_n, sensitivity analysis is performed against $K_n = 34, 36, \ldots, 48, 50$. The results are shown in Fig. 8.

As seen, Model (A) and Model (B) provide similar average waiting time results. However, the real performance of timetables using results from Model (A)—with capacity constraints—has to be tested using simulation. In this case, Model (B) is always superior to the simulation using timetables obtained from Model (A), especially when the number of services is limited (less than 45). When $K_n = 34$, the real average waiting time is $\frac{4.51}{2.64} - 1 = 70.8\%$ higher than optimal result from Model (B).

These results indicate that the capacity constraints effect is substantial when number of services is limited. When number of services increases to 44, Model (B) does not produce different results from model (A) since capacity constrains are not a factor any more.

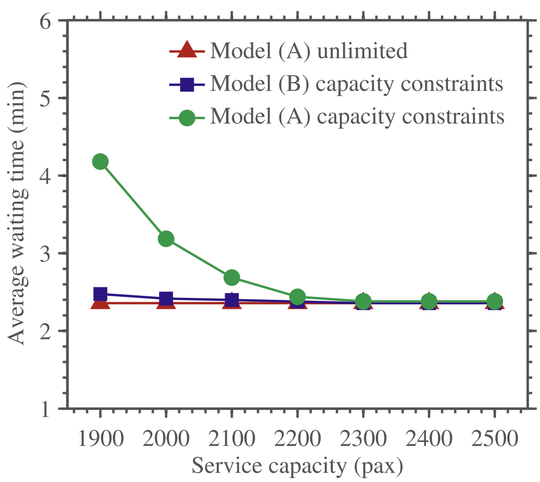

Fig. 9. Balancing service capacity and timetable.

5.3. Balancing capacity and timetable

As with many services, there is also a balance of service capacity. Sensitivity analysis is conducted with $K_n = 36$ (same as the peak/off-peak timetable) against $CAP = 1900, 2000, \ldots, 2500$. The results are shown in Fig. 9.

Without considering capacity constraints, the ideal Model (A) always gives the best results with an average waiting time of 2.36 min. Model (B) produces slightly more waiting time than the ideal results. However, the simulated Model (A) provides quite different results than the ideal model. Owing to limited capacity, simulation on the obtained timetable from Model (A) performs poorly with less service capacity. In this case, average waiting time of timetable (A) with $CAP = 1900$ is $\frac{4.18}{2.47} - 1 = 69.2\%$ higher than timetable (B).

Taken together, in real operation, Model (B) always performs better than (A), particularly when resources, such as number of services and services capacity, are limited. Rather than providing the optimal timetable, Model (A) actually provides the benchmark for timetable design. On the other hand, as can be seen in Fig. 6(b), although the services occupancy may exceed *CAP*, Model (A) shows what the crucial time is for operators to ramp up train capacity (by removing seats or setting up foldable seats).

6. Conclusion and discussion

The emergence of smart card-based automated fare collection system, as implemented in Singapore, enables us to better understand transit demand variation in a more detailed way. In this paper, we propose three models for demand sensitive timetable design: the first model leaves train capacity out of account while the second allows for limited train capacity.

Both models are built on the concept of equivalent time (interval), which discretizes continuous time into discrete values. Although the models seem to provide better results if the equivalent interval is shorter, the computation time and accuracy of demand estimation will be important factors for application. In order to provide applicable strategy for our case, the length of equivalent interval is chosen as $\Delta\tau = 1min$. In addition, it also determines the validity of assumption A2, since demand will change drastically if $\Delta\tau$ is sufficiently small. The problem is also crucial for transfer stations where passengers arrive in batch. In this case, the synchronization between different services may become more crucial. We use average waiting time as a measure of passenger cost without penalizing left-behind passengers; however, left-behind passengers might be more unsatisfied than passengers boarding the first coming train, given the same waiting time. Therefore, an additional penalty can be applied in future analysis to consider the cost of dissatisfaction.

Most developed metro systems are represented by well-connected networks, rather than a single track (on which this study has focused). For a simple network containing two intersecting tracks, we can still apply the concept of equivalent time by calculating offset times against the point of intersection (e.g. the transfer station) on each track and account for a corresponding transfer cost. In a more complex network with multiple transfer stations, though one may give priority to the main track, the strategy of applying equivalent time need to be adjusted accordingly. Our future study will consider the network case.

Considering that new timetables are designed using historical demand data, we cannot ascertain how much new demand (in coming weeks, for example) will differ from historical and how passengers will adjust their traveling behavior given a new timetable. Taking advantage of a continuous stream of smart card data, we can try to maintain demand homogeneity by frequent updates, such as using demand from previous week to design a timetable for the current week. We will also try to explore passenger response to a new timetable as a future research topic.

The proposed models are evaluated by simulating an MRT case service in Singapore; sensitivity analysis is conducted based on a minor case. Examining our analyses together, Model (B) provides exact optimal dynamic timetables under capacity constraints, whereas Model (A) shows significant potential for offering a dynamic timetable with dynamic capacity. Although the timetable (2) performs not as well as (3) and (4) with current demand, it shows some advantages such as being easier to operate and more understandable to passengers. All these optimal solutions can be used as benchmarks to measure current service levels. A major limitation of this study is the absence of crew team scheduling, which is critical for calculating transit operators' cost. On the other hand, given the principle of dynamic timetables, optimal train schedules will no longer involve regular intervals, but will vary over time. The irregular headway/arrival times would probably have an impact on expectations of some punctual passengers, resulting in a potential barrier to implementation. Nevertheless, given the more uniform arrival times across a larger population, the dynamic timetable has substantial potential to reduce total waiting time.

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